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ACTIVATED SLUDGE SYSTEMS. KINETICS OF THE STEADY AND TRANSIENT STATES

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ACTIVATED SLUDGE SYSTEMS

KINETICS OF THE STEADY AND TRANSIENT STATES

by

David W. Eckhoff
and
David Jenkins

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Sanitary Engineering Research Laboratory
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KEY TO SYMBOLS

Symbol	Equivalent	Terminology	Dimensions
COD		Chemical oxygen demand	
D	$\frac{F}{V} = \theta^{-1}$	Hydraulic dilution rate	T^{-1}
$\overline{D}$		Average hydraulic dilution rate	T^{-1}
D_c	$\frac{Fx_2}{Vx_1} = \theta_c^{-1}$	Cell dilution rate	T^{-1}
F		Process flow rate	L^3T^{-1}
$\left(\Delta F\right)_{syn}$		Free energy change during synthesis of cell material (k-cal/mole of carbon)	
$\left(\Delta F_f\right)_{C.M.}$		Free energy of formation of cell material (k-cal/mole of carbon)	
$\left(\Delta F_f\right)_{CO_2}$		Free energy of formation of carbon dioxide (k-cal/mole of carbon)	
$\left(\Delta F_f\right)_{H_2O}$		Free energy of formation of water (k-cal/mole of carbon)	
$\left(\Delta F_f\right)_{sub}$		Free energy of formation of substrate (k-cal/mole of carbon)	
$\left(\Delta F_{ox}\right)_{C.M.}$		Free energy of oxidation of cell material (k-cal/mole of carbon)	
$\left(\Delta F_{ox}\right)_{sub}$		Free energy of oxidation of substrate (k-cal/mole of carbon)	
G	$(\mu)_{S.S.}$	Steady state growth rate	T^{-1}
$H(c,t)$		Sawtooth function of period 2c	
K_s		Saturation constant (Monod equation)	ML^{-3}
N		Total number of moles of cell carbon in the system	
P		Metabolic product	M
S		Growth-limiting substrate	M
T	$t - 2(k+2)c$	Time, referenced to beginning of sawtooth function	T
V		Volume of reactor	L^3
X		Cell material	M

KEY TO SYMBOLS (Continued)

Symbol	Equivalent	Terminology	Dimensions
Y	$-\frac{dX}{dS}$	Cell yield	
Y_e		Effective or apparent cell yield	
Y_o		True cell yield	
Y_z	$-\frac{dX}{d(COD)}$	Cell yield based on COD removal	$\frac{M(\text{cell material})}{M(COD)}$
c_i	$\frac{(w_i)_o}{z_o}$	Fraction of feed COD consisting of constituent i	
c_o		Fraction of feed COD degraded	
c_1		c_2Y	
c_2		A coefficient	
k_i		Reaction rate coefficient for constituent i (based on total biomass concentration x_1)	$M^{-1}L^3T^{-1}$
k_o		Overall rate coefficient (based on residual substrate COD and total biomass concentration x_1)	$M^{-1}L^3T^{-1}$
k_e		True endogenous metabolism rate coefficient	T^{-1}
k_d		Apparent endogenous metabolism rate coefficient	T^{-1}
k_o'		Apparent overall rate coefficient	$M^{-1}L^3T^{-1}$
k_{ij}'		Reaction rate coefficient for constituent i (based on metabolism by microbial species j of reactor concentration x_j)	$M^{-1}L^3T^{-1}$
p_1		Concentration of metabolic products in reactor	ML^{-3}
$(p_1)_o$		p_1 at time zero	ML^{-3}
q	$\frac{F(s_o - s_1)}{Vx_1}$	Specific substrate removal rate	T^{-1}
q_{max}		Maximum value of q	T^{-1}
q_z	$\frac{F(z_o - z_1)}{Vx_1}$	Specific COD removal rate	T^{-1}
$<q_z>$		Psuedo-asymptotic value of q_z following step transient	T^{-1}
s		Concentration of growth-limiting substrate	ML^{-3}
$(s)_{ads}$		Concentration of adsorbed growth-limiting substrate	ML^{-3}

KEY TO SYMBOLS (Continued)

Symbol	Equivalent	Terminology	Dimensions
s_o		Feed concentration of growth-limiting substrate	ML^{-3}
s_1		Reactor concentration of growth-limiting substrate	ML^{-3}
$\langle s_1 \rangle$		Pseudo-asymptotic value of s_1 following step transient	ML^{-3}
$(s_1)_o$		s_1 at time zero	ML^{-3}
$(s_1)_\infty$		s_1 at infinity (under conditions of step transient)	ML^{-3}
$(s_1)_{o-}$		s_1 prior to pulse transient	ML^{-3}
$(s_1)_{o+}$		s_1 immediately after pulse transient	ML^{-3}
s_2		Effluent concentration of growth-limiting substrate	ML^{-3}
t		Time	T
w_i		Concentration of constituent i as COD	ML^{-3}
$(w_i)_o$		Feed concentration of constituent i as COD	ML^{-3}
$(w_i)_1$		Reactor concentration of constituent i as COD	ML^{-3}
x		Concentration of cell material (biomass)	ML^{-3}
x_o		Feed concentration of cell material	ML^{-3}
x_1		Reactor concentration of cell material	ML^{-3}
$(x_1)_o$		x_1 at time zero	ML^{-3}
$(x_1)_\infty$		x_1 at infinity (under conditions of step transient)	ML^{-3}
x_2		Effluent concentration of cell material	ML^{-3}
x_j		Concentration of cell material of microbial species j	ML^{-3}
$(x_j)_1$		Reactor concentration of cell material of microbial species j	ML^{-3}
z		COD (concentration)	ML^{-3}
z_o		Feed COD	ML^{-3}

KEY TO SYMBOLS (Continued)

Symbol	Equivalent	Terminology	Dimensions
z_1		Reactor COD	ML^{-3}
$\langle z_1 \rangle$		Pseudo-asymptotic value of z_1 following step transient	ML^{-3}
θ	$\frac{V}{F} = D^{-1}$	Mean reactor hydraulic residence time	T
θ_c	$\frac{Vx_1}{Fx_2} = D_c^{-1}$	Mean reactor cell residence time (as defined)	T
ϵ		Sorption coefficient	$M^{-1}L^3$
μ	$\frac{1}{X}\frac{dX}{dt}$	Specific growth rate	T^{-1}
μ_{max}		Maximum value of μ	T^{-1}

I. INTRODUCTION

NEED FOR STUDY

Biological waste treatment methods are currently the most economical and widely-used means of removing soluble and colloidal organic constituents from aqueous waste streams. The efficiency of these treatment processes has been determined quite precisely in the laboratory but such laboratory experiments on synthetic substrates under steady flow conditions have limited value in predicting plant-scale performance. As the demand for water of a consistently high quality increases it is becoming necessary to determine the factors that influence the practical performance of biological waste systems.

This need is doubly critical when it is realized that the efficient operation of secondary treatment processes may not only influence effluent quality from these systems *per se* but also influence the functioning of subsequent tertiary treatment processes.

In order to determine the way in which secondary treatment processes function in the field one of the important factors that must be studied is the effect of variable or transient loading conditions. The significance of transient operation cannot be overemphasized since in the majority of waste treatment plants, transients are the rule rather than the exception. The diurnal variation of domestic waste flows is characteristically from 50 percent to 150 percent of the average and loading rates are further distorted since sewage strength variations closely follow the diurnal flow pattern. Thus it is not uncommon for the organic loading rate imposed on a domestic waste treatment process to vary by a factor of 20 or more within a period of one day. Moreover, the organic load often changes from minimum to maximum in about four hours during the morning.

Biological processes treating industrial discharges are often subject to transient conditions even more severe than domestic waste treatment systems. Batch operation of industrial processes, as well as product conversion and shutdown, can result in slug discharges which presently can only be accommodated by the use of large flow-equalization basins and conservative treatment plant designs.

OBJECTIVES OF STUDY

The objectives of the research can be stated as follows:

1. To develop a mathematical model based on a combination of microbiological growth kinetics and process materials balances that quantitatively describe the steady and transient state behavior of the activated sludge process.

2. To test the mathematical model by conducting activated sludge experiments at steady and unsteady state on a laboratory scale using a synthetic substrate. The study of transient behavior is to be confined to short-term single and fluctuating transients of organic load.

3. To conduct limited studies of a pilot-scale activated sludge system treating domestic sewage in order to determine the applicability of the model to a practical situation.

4. To formulate predictions, from previous studies, concerning the effects of short-term transients on effluent quality from the activated sludge process.

APPROACH RATIONALE

In general the currently-used biological waste treatment schemes have evolved through the extension of successful practices. Numerous examples can be cited, e.g., step aeration, step loading, contact stabilization, etc. in which the flow scheme of the activated sludge process has been successfully modified as a result of plant-scale experiments conducted on an empirical basis.

While such an approach to process modification speaks well of the ingenuity and versatility of the engineer and plant operator, it nevertheless is open to criticism on the count that empiricism is usually an inefficient method of advancing technology. For the activated sludge process the principles of microbiological growth in continuously-fed systems must be combined with materials balance and unit process concepts to provide such a fundamental basis — a basis that should allow the rational development of effective design parameters and operation procedures as well as the accurate prediction of process performance.

Using this rationale, the investigation first developed a mathematical model describing the activated sludge process for steady and transient state conditions and then tested the model under laboratory and pilot-plant conditions.

ACKNOWLEDGMENTS

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The authors express their appreciation to Mrs. Hazel Gaffey and Mr. Joram Diamant for their technical assistance and to Mr. Arnold B. Menar for his aid in the monitoring of the pilot plants.

II. LITERATURE REVIEW

INTRODUCTION

The activated sludge process is a microbiological enrichment culture in which a rather extensive ecological system develops. An array of organisms both coexists and competes in the metabolism of nutrients present in the waste stream. There are, however, several restrictive conditions which tend to favor the establishment of a dynamically balanced flora. The first is that the culture is continuously inoculated with microorganisms by the incoming waste. Secondly, even though short-term variations in the composition of waste streams are common, long-term variations are not frequently encountered. Finally, the successful operation of the activated sludge process depends on the retention of microorganisms within the system. Gravity separation in settling basins and recycle of settled organisms to the aeration tank constitute the most common method of maintaining the organisms in the system for periods longer than the hydraulic residence time. Hence those organisms having characteristics which make them amenable to sedimentation tend to predominate. The culture is therefore "enriched" because relatively rigid environmental conditions are maintained, and the desired organisms are selectively encouraged by the chosen technique of cell recycle.

The activated sludge process differs from the classical enrichment culture because in the former both nutrients and diluting liquid are continuously added to the reactor, or culture vessel. In this respect it bears a strong resemblance to continuous microbial cultivation techniques which are only distinguished from continuous flow chemical reactor operations by the autocatalytic and biological nature of the reactions. It therefore appears logical to examine continuous culture theory to obtain a description of the activated sludge process.

The four basic factors involved in the microbiological growth model — growth rate, substrate concentration relationships, cell yield, and endogenous metabolism rate are discussed under the heading 'Kinetic Model of Microbial Growth.' These basic concepts of microbiological growth have been used in a model that describes the activated sludge process. (Cf. Chapter III.) A discussion of some of the results obtained by previous investigators who have applied microbial growth models to the prediction of activated sludge process performance is described under 'Engineering Investigations.'

The transient state model developed in this study accounts for the rapid initial uptake of organic matter by activated sludge. Previous work on this phenomenon is reviewed under the heading 'Initial Uptake of Organic Matter by Activated Sludge.'

As a result of the complex nature of domestic sewage, it is not possible to identify and make a specific measurement of growth-limiting substrate. Consequently, a nonspecific measurement of organic substrate — that of COD — must be employed. The nonspecific nature of this measurement introduces problems in data interpretation which are discussed under the section entitled 'Residual Nondegradable COD.'

KINETIC MODEL OF MICROBIAL GROWTH

Specific Growth Rate

One of the most important characteristics of a microbiological culture is the relationship of the specific growth rate of the microorganisms to the concentration of growth-limiting substrate. In general, specific growth rates are stimulated by increases in the growth-limiting substrate concentration. A widely accepted general expression for this relationship is the Monod equation [1]:

$$\mu = \mu_{max}\left(\frac{s}{K_s + s}\right) = \frac{1}{X} \cdot \frac{dX}{dt} \qquad (1)$$

where: μ = specific growth rate, (T^{-1})

s = growth-limiting substrate concentration, (ML^{-3})

K_s = saturation constant, (ML^{-3})

μ_{max} = maximum specific growth rate, (T^{-1})

X = mass of microorganisms in system, (M)

t = time, (T).

Monod found that this equation satisfactorily described the growth of pure bacterial cultures at the expense of certain organic substrates when all other essential growth factors were in surplus. At low substrate concentrations the specific growth rate is first order with respect to substrate concentration, whereas at high substrate concentrations μ approaches a limiting value, μ_{max}.

Equation 1 was developed for cultures in which the limiting nutrient was the carbon and energy source, but its validity has been demonstrated when other single nutrients or energy sources were limiting. Its application to mixed substrate—mixed population systems is still open to question because it is often impossible to identify the growth-limiting nutrient.

Other growth rate—substrate relationships have been proposed, and indeed in certain environmental conditions the Monod equation is not applicable; however Equation 1 has a rather formidable historical basis and is the most generally accepted relationship.

Cell Yield

Cell yield, Y, has no restricted definition although its most common designation is "mass of cells produced per unit mass of substrate consumed." Monod [1] found mass yields to be constant for each organism-substrate combination examined but varied between different organisms and for the same organism grown on different substrates. These observations led Monod to postulate that for a given organism-substrate combination, the yield is a constant. Later more detailed work by Herbert [2] suggested that yield and many other physiological properties of microorganisms are functions of the specific growth rate. Herbert concluded that the specific growth rate was the most important single parameter of microbiological systems.

Assuming that cell yield can be defined as "mass of cells produced per unit mass of substrate consumed," then the rate of substrate metabolism is related to the specific growth rate via the constant of proportionality, Y, i.e.,

$$Y = - dX/dS \qquad (2)$$

where S = mass of substrate, (M).

Hence:

$$\begin{aligned} - dS/dt &= (Y^{-1})\, dX/dt = \mu X/Y \\ &= (X/Y)\, \mu_{max} \left(\frac{s}{K_s + s}\right). \qquad (3) \end{aligned}$$

Endogenous Metabolism

The phenomenon of endogenous metabolism, or endogenous respiration, is displayed by all living organisms. In the strictest sense it is the quantity of cellular matter which is sacrificed through metabolism and energy production for purposes of cell maintenance. Dawes and Ribbons [3] concluded that many of the reported values for endogenous metabolism rates include rates of death and decay of cellular meterial. They reason that since the observer is attempting to measure a quantity of cell mass that has been destroyed, any mechanism resulting in a loss of cell mass must necessarily increase the apparent endogenous metabolism rate, unless special precautions are taken to isolate individual contributions to the overall effect of mass decrease.

Although Herbert's work [2] indicates that the apparent endogenous metabolism rate for a microbiological system would not be a fixed quantity, the most common assumption is that, under moderately variable environmental conditions, the rate coefficient k_d is a constant.

ENGINEERING INVESTIGATIONS

Kinetic Models

One of the first kinetic studies of the activated sludge process was made by Garrett and Sawyer [4]. In laboratory-scale experiments these workers developed continuous cultures on synthetic wastes composed of glucose, peptone, and inorganic nutrients. It was assumed that the residual growth-limiting substrate could be represented by the effluent soluble 5-day BOD. Experimental results suggested that a first-order relationship existed between effluent soluble BOD and sludge growth rate at low BOD concentrations. At high soluble effluent BOD concentrations these workers found that the sludge growth rate was zero order with respect to BOD. They therefore proposed a discontinuous, two-phase model relating sludge growth rate to effluent soluble BOD. This model was later adopted and used extensively by Eckenfelder and McCabe [5]. Garrett and Sawyer [4] also noted that the flora which developed on their synthetic substrate was not flocculant like a typical activated sludge but consisted of dispersed single cells.

Gram [6] studied the activated sludge process over a wider range of specific substrate metabolism rates than Garrett and Sawyer; he used laboratory units treating a synthetic waste made up from skim milk. Gram postulated a Langmuir-type surface adsorption of the substrate followed by a first-order reaction. Using BOD as a measure of substrate concentration, his results indicated that a limiting substrate removal was approached at higher substrate concentrations. Gram, like Garrett and Sawyer, was plagued by dispersed growth cultures and he resorted to the continuous feeding of both stock-activated sludge and substrate without secondary sedimentation and sludge recycle. This technique solved the problem of organism washout, but created a situation somewhat removed from the conventional waste treatment flow scheme.

More recent work by Washington _et al_. [7] conducted with pure cultures of microorganisms grown on single organic substrates was related to biological waste treatment by using COD determinations to evaluate residual substrate concentrations. These workers proposed the following specific growth rate equation:

$$\mu = \mu_{max} \left(\frac{s - A}{s - B} \right). \qquad (4)$$

The terms A and B, which account for the presence of nondegradable metabolic end products in the effluent, were not related to other process parameters by these authors.

Wilson [8], in a series of experiments with batch respirometers and rolling-tube filters, attempted to assess the validity of either the Monod equation or the Garrett and Sawyer [4] two-phase model for mixed culture systems by using acetic acid, phthalic acid, and peptone as substrates. Using soluble BOD and COD for substrate determinations, a general conformance to a Monod type relationship was evident from his data. The saturation constant for peptone was 187 mg/ℓ expressed as BOD.

Tench and Morton [9] conducted extensive batch experiments to ascertain the value of the saturation constant for aerobic biological waste treatment systems and found that for constituents characteristic of domestic sewage, K_s was greater than 250 mg/ℓ expressed as BOD. This work and that of Wilson described above suggest that for most cases Equation 1 can be written:

$$\mu \simeq \mu_{max} \left(\frac{S}{K_s}\right) . \qquad (5)$$

Eckenfelder [10] has gathered operating data from a diversified group of activated sludge plants and shown that BOD removal efficiency can be related to the product of cell mass and hydraulic residence time. This conclusion is implicit in Equation 27 of the kinetic model developed later in this report assuming that BOD is representative of the growth-limiting substrate. Anaerobic fermentation research in this Laboratory [11,12] also points to the general applicability of the Monod model to mixed culture—mixed soluble substrate systems.

Cell Yield

The early work of Heukelekian *et al.* [13] showed that the production of sludge could be empirically related to the BOD applied by the following equation:

$$A = 0.5\ B - 0.055\ S \qquad (6)$$

where: A = mass of sludge solids produced per day, (MT^{-1})

B = mass of substrate BOD applied per unit mass of sludge solids per day, (T^{-1})

S = mass of sludge solids in system, (M).

Since the residual BOD values in their experiments were small, the parameter B is nearly equivalent to the specific substrate removal rate, and the equation is approximately equivalent to the theoretical relationship derived later in this report (Equation 30). Thus, Heukelekian's data indicate a cell yield of 0.5 lb sludge solids per lb BOD.

Gram [6] developed an expression similar to Equation 6 and found the sludge yield to be 0.53 lb sludge solids per lb BOD removed.

Data from field studies by Hopwood and Downing [14] have been transposed and are shown in Figure 1. The slope of the fitted line gives a yield value of 0.97 lb sludge total solids per lb BOD applied.

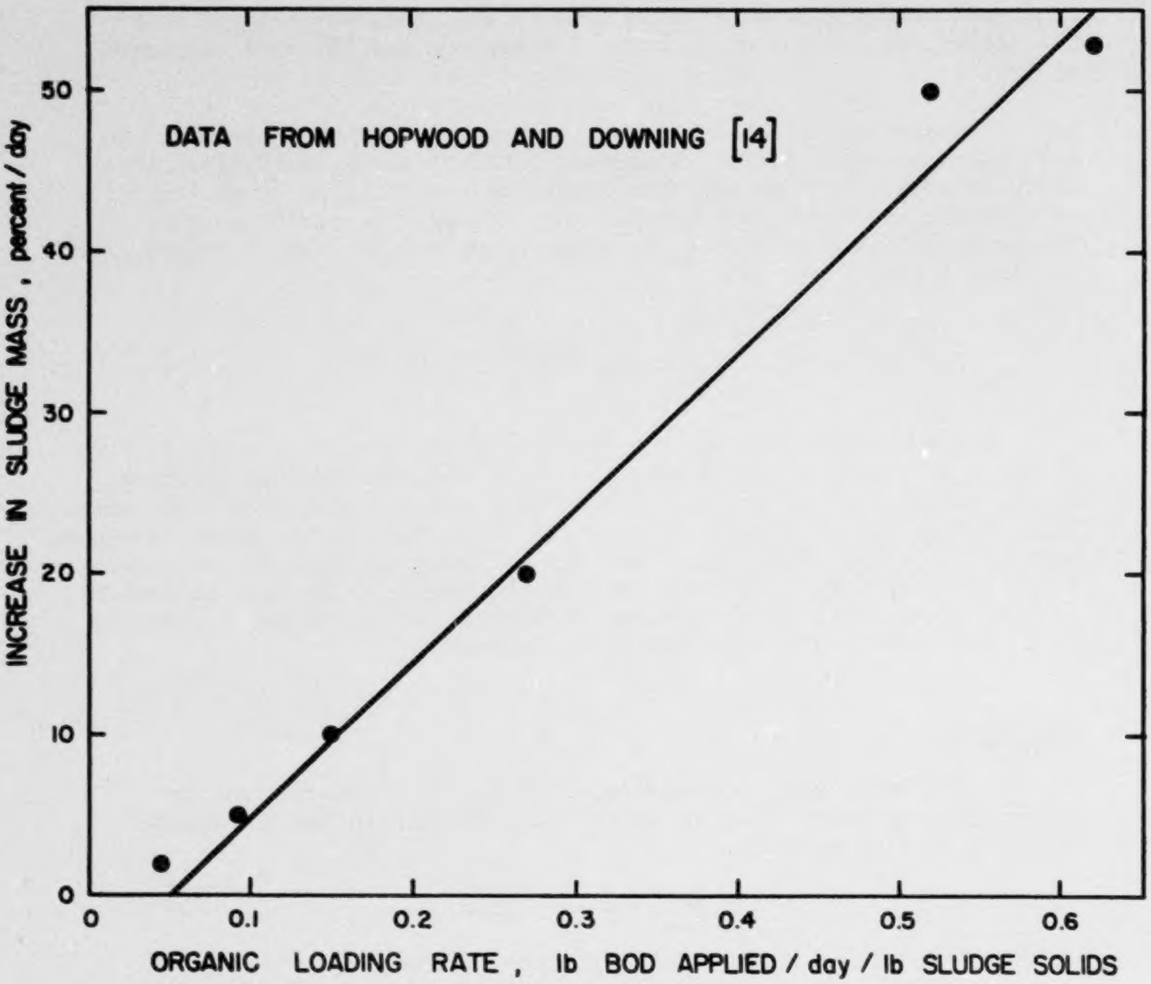


FIGURE I. RESULTS OF BRITISH FIELD STUDIES

These and other data collected under less well-controlled conditions point to the general conclusion that cell yields based on BOD are extremely variable and highly substrate dependent.

Bauchop and Elsden [15] and Siegel and Clifton [16] conducted experiments with pure cultures of bacteria having known metabolic pathways and concluded that cell yields are a function of adenosine triphosphate (ATP) formation, and that each situation must be individually considered.

In a later publication Servizi and Bogan [17] stated that, from an engineering standpoint, the amount of ATP formation can be estimated from a consideration of the free energy changes taking place during metabolism of a substrate. These authors further stated that in aerobic biological processes the principal reaction involving large free energy changes is the total oxidation of the organic substrate and therefore the thermodynamics of this oxidation should control synthesis of cell material. Servizi and Bogan demonstrated that the free energy change involved in the total oxidation of organic compounds was approximately proportional to the quantity of oxygen required for the oxidation of the compound. Thus these authors postulated that cell yields should be approximately proportional to the theoretical oxygen demand of the organic substrate, a quantity that can be readily measured by the COD procedure. Although Servizi and Bogan did not develop this relationship rigorously, it has been possible to prove its validity (see Appendix A).

In discussions of the work of Servizi and Bogan both McCarty [18] and Hetling [19] have argued that biological energy transfer efficiencies are extremely variable and depend on a specific organism-substrate combination (as previously reported by Monod [1], Bauchop and Elsden [15], and Siegel and Clifton [16]). However, it may be argued that for the mixed populations encountered in biological waste treatment a relatively constant overall energy transfer efficiency should be ensured. Indeed the work reviewed below indicates that for mixed population systems, COD values provide a more rational basis for estimating cell yields than do BOD determinations.

Porges *et al*. [20] studied the stoichiometry of biological oxidation and sludge synthesis by conducting experiments with lactose and casein, the two principal constituents of skim milk. A summary of their findings is shown in Table I.

TABLE I

CELL YIELDS OBTAINED BY PORGES *et al*. [20]

Constituent	Cell Yield (g VSS/g substrate)
Lactose	113/240
Casein	113/184

Converting these substrate values to COD gives yields of 0.44 g VSS/g COD for both constituents.

Milbury [21] obtained the identical yield of 0.44 g VSS/g COD in his laboratory experiments with continuous flow activated sludge units fed on skim milk.

A direct count of bacterial numbers was employed by Pipes [22] to evaluate the growth potential of several pure and mixed substrates fed to mixed cultures of sewage origin. His results showed a linear relationship between the theoretical oxygen demand (COD) of the substrates and the total number of cells produced.

Washington et al. [7] have tabulated a comprehensive list of COD-based yield data for both pure and mixed culture systems and conclude that mixed cultures and/or complex media give higher yields than pure cultures grown on simple media. It is evident that heterogeneous systems show much less variable yields than pure culture or pure substrate systems, supporting the postulate that the average energy transfer efficiency of mixed microbial cultures is relatively constant. Recent work on full-scale and pilot-scale activated sludge plants treating domestic sewage has indicated that the COD-based yield coefficients for the normally-encountered practical loading rates are quite constant from plant to plant. Thus Jenkins and Menar [23] found a yield coefficient of 0.33 lb VSS/lb COD for a pilot plant activated sludge unit at Richmond, California, while Jenkins and Garrison [24] determined the yield coefficient from the 3-mgd Pomona Water Renovation Plant to be 0.32 lb VSS/lb COD.

Endogenous Metabolism

Since the term endogenous metabolism has been used indiscriminately, one must exercise caution in the interpretation of values of this coefficient reported in the literature. Although there is no conceptual unity concerning the nature of endogenous metabolism, in biological waste treatment research it has been commonly assumed that it occurs at a constant rate. As used in Equation 30, developed later in this report, the phenomenon is either:

1. The sum of all processes which result in the depletion of cell mass, or
2. The cell-mass equivalent of nutrients metabolized for purposes other than cell synthesis.

A summary of the values of endogenous metabolism coefficients which has been obtained from the literature and whose definitions coincide with either of the above concepts is included in Table II.

Kinetic Summary

There have been a number of semikinetic studies made on the various modifications of the activated sludge process. However, most of these studies have not employed continuous flow reactors. For a homogeneous reaction the choice of a continuous flow or batch system is one of overall experimental convenience. However, for a heterogeneous reaction, such as the activated sludge process where the biological catalyst is strongly influenced by both previous and present environmental conditions, the use of batch data for the evaluation of kinetic parameters is open to serious question. Kinetic data from carefully conducted experiments are summarized in Table II. The batch experiments

TABLE II

SUMMARY OF REPORTED KINETIC DATA FOR ACTIVATED SLUDGE SYSTEMS

Author(s)	Substrate	System	Temperature °C	μ_{max} day^{-1}	K_s, mg/ℓ	Y	k_d day^{-1}	Notes
Gram [6]	Skim milk	Continuous flow	-	(2.4)[a]	100 BOD_5	0.53 SS/BOD_5	0.045	$\hat{\mu} = Y\hat{q}$ one pass dispersed growth systems
Porges *et al*. [20]	Skim milk, lactose, casein	Batch	-	-	-	(0.44)[a] VSS/COD	0.08	Obtained from accurate stoichiometry
Garrett and Sawyer [4]	Peptone	Batch	20°	5.0	-	-	-	Dispersed growth systems
	Peptone	Batch	30°	6.2	(109)[a] BOD_5	-	-	
	Glucose	Batch	10°	1.9	-	-	-	
	Glucose	Batch	20°	4.3	-	-	-	
	Peptone and glucose	Batch	20°	5.0	-	-	-	
	Peptone and glucose	Batch	30°	7.9	-	-	-	
	Averages	Batch	10°	-	-	0.53 VSS/BOD_5	-	
	Averages	Batch	20°	-	-	0.43 VSS/BOD_5	-	
	Averages	Batch	30°	-	-	0.38 VSS/BOD_5	-	
Tench and Morton [9]	Peptone	Batch	-	-	296 BOD_5	-	-	
	Glucose	Batch	-	-	133 BOD_5	-	-	
	Sewage	Batch	-	-	259 BOD_5	-	-	
McWhorter and Heukelekain [25]	Glucose: NO_3^-	Batch	- -	- -	- -	33.7 % (w/w) (0.316)[a] VSS/COD	- -	Assumed 90% COD removal, 90% volatile material
Milbury [21]	Skim milk	Continuous flow	20°	2.9	110 COD	0.43 VSS/COD	-	
Jenkins and Menar [23]	Domestic sewage	Continuous flow	~20°	-	-	0.33 VSS/COD	0.001	
Jenkins and Garrison [24]	Domestic sewage	Continuous flow	~20-25°	-	-	0.32 VSS/COD	~0	

[a] Modified or calculated values enclosed in parentheses.

in Table II were conducted under conditions that did not affect the results or additional experiments were made to determine the effect of batch operation on the data.

INITIAL UPTAKE OR ORGANIC MATTER BY ACTIVATED SLUDGE

Introduction

The rapid initial uptake of nutrients by activated sludge has been the subject of numerous investigations. Conflicting reports concerning the nature of the phenomenon are common and numerous terms have been employed for its description, some of which are tabulated in Table III. Despite the importance of this phenomenon in biological waste treatment, there is no general agreement on either its mechanism(s) or kinetics.

Mechanism

The earliest investigators of the phenomenon of rapid uptake attempted to explain their results in terms of ion exchange or colloidal attraction (Heukelekian [26] Katz and Rohlich [34]. Buswell [27] later concluded that the biological activity of the sludge organisms was essential for rapid removal since it was not possible to demonstrate the presence of adsorbed material on activated sludge. Heukelekian [26] and Ingols [35] described experiments on the assimilation of various

TABLE III

TERMS USED TO DESCRIBE RAPID INITIAL UPTAKE
OF SUBSTRATE BY ACTIVATED SLUDGE

Authors	Terms
Heukelekian [26]	Clarification Stage
Gram [6]	Adsorption
Porges *et al*. [28]	Nonoxidative Accumulation
Eckenfelder and O'Connor [29]	Adsorption Biosorption } Initial Removal
McCabe [30]	Adsorption
Eckenfelder [31]	Absorption
Wilson [8]	Rapid Removal
Gaudy and Engelbrecht [32]	Initial Steps
Weston and Stack [33]	Initial Removal
Katz and Rohlich [34]	Adsorption

organic and inorganic materials. Unfortunately the former were restricted to soluble organics and measurements of the latter (kaolin and infusorial earth) were made using nephelometry. The removal of the clays apparently followed the "laws" of adsorption, although a quantitative assessment could not be made of the phenomenon. The uptake of the soluble organic materials (glucose, peptone, casein, and egg albumen) "...could not be ascribed to adsorption" [26].

Wuhrmann [36] commented on experimental data obtained by several other investigators as follows:

(a) The high initial rate can be reasonably demonstrated only by considering BOD removal from wastes containing appreciable quantities of suspended material.

(b) At that time (1956) it had not been possible to demonstrate the presence of adsorbed soluble organic material on sludge flocs.

Wuhrmann concluded that "...adsorption does not explain the rapid removal of soluble organic compounds from wastes by activated sludge." Wuhrmann presented time-concentration curves for various soluble organic materials in batch activated sludge units. While some results apparently can be explained by first-order reactions, and others follow a linear time-concentration relationship, any further elucidation of the kinetics of initial uptake is not presently possible.

Batch activated sludge experiments on skim milk substrate by Porges _et al_. [28] demonstrated that large amounts of substrate were assimilated by the sludge organisms in a nonoxidative pathway (about 60 percent of the total COD after 1 hr at 20° C). The material could not be eluted by washing the sludge, and experiments showed that large quantities of glycogen were present in the activated sludge organisms shortly after batch feeding.

Mathematical and Kinetic Models

Katz and Rohlich [34] suggested a quantitative description of initial uptake using adsorption as the rate-limiting step in their model. The concentration of nutrients at the bacterial surface was related to the equilibrium concentration in solution by the expression:

$$\frac{X'}{M} = KC_e \tag{7}$$

where: X' = initial quantity of adsorbate (BOD)

M = quantity of adsorbent (sludge)

C_e = equilibrium concentration of BOD in solution

K = a coefficient.

Examination of the experimental data [34] indicates that K is not constant but varies with the concentration of adsorbent approximately as follows:

$$K \simeq \frac{K'}{C_m} = \frac{K'}{M/V}$$

where: C_m = concentration of adsorbent (sludge)

V = volume of system

K' = a coefficient.

Combining these equations we have

$$\frac{X'/V}{M/V} \simeq \left(\frac{K'}{M/V}\right) C_e$$

or

$$\frac{C_e}{C_o} \simeq \left(\frac{1}{K'}\right) = \text{a constant} \qquad (8)$$

where C_o = initial BOD concentration. Equation 8 states that the equilibrium adsorbate concentration is independent of the adsorbent concentration — a statement that cannot be reconciled with either the Katz and Rohlich [34] hypothesis or any equilibrium theory of adsorption.

Eckenfelder [31] and Weston and Stack [33] independently proposed the same relationship for initial uptake of BOD:

$$L_{ri} = bL_oS_o$$

where: L_{ri} = initial BOD removal

L_o = initial BOD applied

S_o = sludge concentration

b = a coefficient.

No unit of time is included in this equation and it is thus inferred that the equation represents the situation after some fixed time interval.

Rearranging the expression, we obtain

$$L = L_o \left(1 - bS_o\right) \qquad (9)$$

which Eckenfelder [31] stated is in agreement with the findings of Katz and Rohlich [34]. However, neither of the above papers [31,34] present data that conclusively substantiate this claim.

Gram [6] hypothesized that a Langmuir-type adsorption isotherm describes the kinetics of BOD uptake by activated sludge, thus

$$Y_{ads} = ax \left(\frac{cy}{1 + cy}\right) \qquad (10)$$

where: Y_{ads} = concentration of "adsorbed" BOD

a = the maximum "adsorptive capacity" of sludge (mg BOD/mg sludge solids)

c = a coefficient

y = equilibrium concentration in the liquid phase

x = concentration of sludge solids (MLSS).

When y is large ($cy > 1$), $Y_{ads} \simeq ax$, and

$$Y \simeq Y_o - ax \qquad (11)$$

where Y_o = initial BOD concentration. Gram [6] further proposed that oxidation is proportional to "adsorbed BOD," which for his experiments consisted of skim milk. Although his experimental data appear to substantiate this choice of model, the work of Porges _et_ _al_. [28] tends to negate this assumption.

In a later publication, Eckenfelder and O'Connor [29] presented the following relationship to represent the initial uptake of BOD:

$$\frac{dL_i}{L_i} = - K_i dS$$

where: L_i = the BOD remaining which is amenable to rapid removal in a specified time

S = sludge concentration

K_i = a coefficient.

On integration this expression yields

$$L_i = L_{io} e^{-K_i S} \tag{12}$$

where L_{io} = the maximum BOD amenable to rapid removal in the specified period of time. The quantity L_{io} is quite subjective, depending upon the waste, the sludge, and the analyst.

Equations 9, 11, and 12, although seemingly incompatible, are in fact strikingly similar. They appear to be different principally because of inconsistent terminology.

Thus let us first consider the equations developed by Eckenfelder [31] and Weston and Stack [32] (Equation 9).

Let

$$b = \frac{a}{L_o} \tag{13}$$

where a is equivalent to the adsorptive capacity proposed by Gram [6]. This is permissible, since the authors [31,33] did not specify that b is constant. Substituting Equation 13 into Equation 9 we obtain

$$L = L_o - aS_o \ . \tag{14}$$

Next, considering the relationship given by Eckenfelder and O'Connor [29], let

p = fraction of initial BOD not amenable to rapid removal in the specified time period.

Then $L_{io} = (1 - p)\, L_o = L_o - pL_o$

$L_i = L - pL_o$ = total BOD remaining minus BOD not amenable to initial removal

where L_o = total initial BOD

L = total BOD remaining.

Substituting these values into Equation 12 yields

$$\left(L - pL_o\right) = \left(L_o - pL_o\right) e^{-K_i S}$$

or

$$L = L_o e^{-K_i S} + pL_o \left(1 - e^{-K_i S}\right). \qquad (15)$$

If the exponentials are approximated by expansion to two terms, i.e.:

$$e^{-x} \simeq 1 - x ,$$

then Equation 15 becomes:

$$L \simeq L_o - L_o\left(K_i S\right) + pL_o\left(K_i S\right)$$

or

$$L \simeq L_o - \left[(1 - p)\ L_o K_i\right] S . \qquad (16)$$

Now it is possible to suggest

$$K_i = \left[\frac{a}{(1 - p)\ L_o}\right]$$

which upon substitution into Equation 16 gives

$$L \simeq L_o - aS . \qquad (17)$$

This derivation shows that there is a great similarity between the expressions relating the rapid uptake of BOD by activated sludge to the equilibrium BOD concentration and the activated sludge concentration (Table IV).

TABLE IV

MODIFIED FORMULAS DESCRIBING INITIAL UPTAKE
OF ORGANIC MATTER BY ACTIVATED SLUDGE

Authors	Modified Formula
Eckenfelder [31]; Weston and Stack [33]	$L = L_o - aS_o$
Eckenfelder and O'Connor [29] . .	$L \simeq L_o - aS$
Gram [6]	$Y \simeq Y_o - ax$

Discussion

The approximate agreement among the equations presented in Table IV must be considered somewhat fortuitous inasmuch as many mechanisms are probably involved in the phenomenon of initial uptake. With the limited data presented [29, 31, and 33] it is not possible to compare the values of the various "adsorption coefficients" and it would be unreasonable to expect them to be of the same order of magnitude. However, the important general implication that can be drawn from the modified expressions is that initial uptake increases almost linearly with the mass of the microorganisms in the system. Only Gram [6] conducted experiments on a nonparticulate substrate. The other investigations were carried out on a variety of wastes which in all probability contained particles which ranged in size from macroscopic to molecular. It is therefore improbable that the same mechanism can describe the uptake of nutrients from such a spectrum of particle dimensions. The most logical assumption is that physical adsorption of the larger particle onto the sludge floc is a dominant mechanism, while assimilation for the formation of storage products or for oxidation comprises the major means of removal of molecular-sized materials. The work of Smallwood [37] supports this assumption. Although the latter mechanism really amounts to a type of absorption (due to the accumulation of intermediates and storage products) it should occur at a slower rate because of intermediate transport and reaction phenomena.

RESIDUAL NONDEGRADABLE COD

Introduction

If COD residuals are to be employed as a measure of growth-limiting nutrient, it must be recognized that certain organic constituents which are resistant to biodegradation may be present in the feed material. In addition, refractory metabolic products may be formed during the metabolism of the principal substrate. While the organic portion of these metabolic end products is readily oxidized in the dichromate COD procedure, these materials do not contribute measurably to the growth of the activated sludge organism. There has only been limited investigation of the nature and significance of these metabolically resistant materials.

General Information

McWhorter and Heukelekian [25] and Rao and Gaudy [38] obtained COD residuals of between 5-15 percent of the initial substrate concentration when batch activated sludge units were fed a synthetic medium containing glucose as the sole source of carbon and energy. Glucose, only traces of which remained in the effluent, did not account for a significant fraction of the residual COD. These studies indicate that a major fraction of the residual COD can be ascribed to materials not present in the feed. Since the formation of residual nondegradable COD showed no dependence on the concentration of sludge, it is possible that the production of nondegradable materials must be substrate linked.

The temporary accumulation of metabolic intermediates was demonstrated in batch experiments by Anderson and Washington [39] who grew *Pseudomonas fluorescens* on glutamic acid C-14 and glucose C-14. By comparing the residual substrate C-14 to the total soluble residual C-14, the existence of soluble intermediates was verified. These intermediates were subsequently metabolized after the primary substrate had been depleted. In the glucose experiments the soluble C-14 obtained after eight hours aeration amounted to five percent of the added C-14.

Porges *et al*. [28] found a residual COD of between 5-7 percent of the feed COD; Bechir and Symons [40] reported very little change in effluent soluble COD values in continuous flow units operated at cell dilution rates between 0.025 and 0.4 day^{-1}; Milbury [21], Tenney *et al*. [41], Behrens and Klappach [42], and Evans and Kite [43,44] all showed an accumulation of residual soluble COD but did not attempt to relate the quantity of this material to their system parameters.

Cassel [45] developed a steady state kinetic model using COD as a substrate parameter in experiments on a skim milk substrate and attempted to account for the refractory residual which amounted to 5.75 percent of the feed COD. This residual was subtracted from the total residual soluble COD to obtain the growth-limiting substrate concentration.

Nature of Metabolic End Products

While microbiological products such as ethanol, lactic acid, and penicillin are important commercial products and numerous industrially important processes for the production of these materials have been studied in great detail, the characterization of the organic end products from the flora of biological waste treatment processes has not received such attention.

Even in the limited work on the residual organic compounds in sewage effluents a considerable fraction of the material has always been unidentified.

Painter *et al*. [46] analyzed the soluble organic constituents of a standard-rate trickling filter effluent and accounted for only 25 percent of the organic carbon (Table V).

Bunch *et al*. [47] compared the soluble constituents of several activated sludge plant effluents (Table VI). Their results, for the Chicago, Ill. and Hamilton, Ohio plants show a ratio of COD to identified organics of between 3.3 and 3.5, suggesting that approximately 50 percent of the soluble organic carbon was unidentified.

TABLE V

ANALYSIS OF SOLUBLE ORGANIC CARBON IN A TRICKLING FILTER EFFLUENT
(Painter *et al*. [46])

Constituents	Carbon (mg/ℓ)
Protein	0.25
Amino Acids	0.06
Carbohydrate	0.24
Soluble Acids	1.65
Anionic Surfactant	1.40
Total Identified	3.60
Total Carbon by Analysis	14.0
Carbon Unidentified	10.4 (74%)

TABLE VI

ANALYSIS OF SOLUBLE COMPONENTS OF ACTIVATED SLUDGE EFFLUENTS
(Bunch *et al*. [47])

Analysis	Concentration (mg/ℓ of indicated material)	
	Chicago	Hamilton
COD	25.3	48.9
Volatile material at 600° C	53	59
Anthrone reactive as glucose	1.3	1.1
Total Kjeldahl nitrogen	0.5	1.7
Protein	1.7	4.8
ABS	2.6	4.6
Ether extractables	1.2	3.2
Tannin, lignin (Folin Dennis reactive)	0.5	0.9

Kinetics of Product Formation

In a previous section mention was made of three microbiological products: ethanol, lactic acid, and penicillin. The kinetics of the reactions leading to these three products are distinctly different; however, the three rate expressions cover a wide range of processes involving the formation of microbiological end products.

Herbert [48] discussed the production of ethanol by *Saccharomyces cerevisiae* cultivated on glucose. The metabolic pathways has been well defined (Embden-Meyerhof), and two molecules of ethanol are produced for every molecule of glucose consumed. In this case the rate law is

$$dP/dt \propto - dS/dt \qquad (18)$$

where: P = mass of product (ethanol)

S = mass of substrate.

The rate of formation of lactic acid by homofermentative organisms grown on glucose in batch culture has been described by Ludeking and Piret [49] as

$$dP/dt = \alpha(dX/dt) + \beta X \qquad (19)$$

where: α, β = fermentation constants

X = mass of organisms.

Equation 19 is identical to Equation 18 except that an additional term has been included to account for metabolite production not associated with growth.

The formation of penicillin by *Penicillium* spp. is entirely independent of growth rate. The production of the antibiotic commences when the organisms reach a specific physiological state. The rate is given by Herbert [48] as

$$dP/dt = KX . \qquad (20)$$

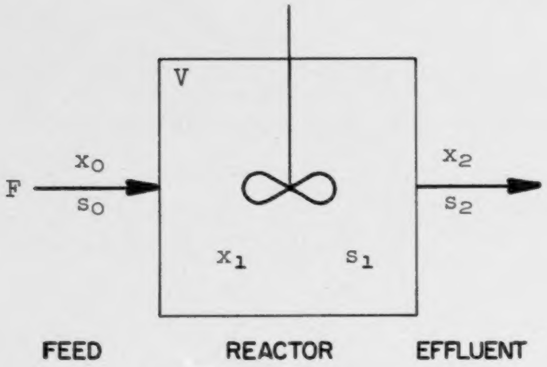
There have been no reported attempts to develop a kinetic equation which describes end product formation in biological waste treatment processes.

III. THEORETICAL RATIONALE

SINGLE ORGANISM—SINGLE SUBSTRATE SYSTEMS

Organism Materials Balance

For a uniformly mixed continuous culture reactor (CSTR) a mass balance for cell material provides an expression for the steady state concentration of growth-limiting substrate in the reactor (Figure 2).



SUBSCRIPT DESIGNATION:

o = Feed
1 = Reactor
2 = Effluent

SYMBOLS:

F = Process Flow Rate
V = Reactor Volume
x = Cell Concentration
s = Substrate Concentration

FIGURE 2. SCHEMATIC DIAGRAM OF UNIFORMLY MIXED CONTINUOUS CULTURE REACTOR

Change = Input - Output + Growth - Endogenous Metabolism

$$d(Vx_1)/dt = Fx_0 - Fx_2 + \mu(Vx_1) - k_d(Vx_1) \qquad (21)$$

where: V = reactor volume, (L^3)

F = process flow rate, (L^3T^{-1})

x_0 = cell concentration in process feed, (ML^{-3})

x_1 = cell concentration in reactor, (ML^{-3})

x_2 = cell concentration in process effluent, (ML^{-3}).

In a system such as activated sludge where no significant mass of cells is fed to the reactor, the cell dilution rate, D_c, can be defined as the mass rate of cells issuing from the system per unit mass of cells in the system.

$$D_c = Fx_2/Vx_1 = Dx_2/x_1 \qquad (22)$$

where D = process dilution rate, (T^{-1}). At steady state Equation 21 becomes

$$D_c x_1 = (\mu - k_d)x_1 \; . \qquad (23)$$

Substituting the Monod equation (1) for μ gives

$$s_1 = \frac{(D_c + k_d)K_s}{\mu_{max} - (D_c + k_d)} \; . \qquad (24)$$

Thus for given microorganism characteristics $(\mu_{max}, K_s, \text{and } k_d)$, s_1 is solely a function of D_c. It can be seen that the important waste treatment objective of reducing the organic waste concentration, s_1, to a low level will be achieved when D_c is small, that is when the discharge of microorganisms from the system is small.

Substrate Materials Balance

The steady state concentration of microorganisms is obtained from Equations 21-24 and a substrate mass balance:

Change = Input - Output - Uptake

$$d(Vs_1)/dt = Fs_o - Fs_2 - \mu Vx_1/Y \qquad (25)$$

where Y = yield = mass of microorganisms formed per mass of substrate utilized. For a continuous stirred reactor (CSTR)

$$s_2 = s_1 \; . \qquad (26)$$

Combining Equations 23 and 25, the steady state value of x_1 can be derived

$$x_1 = \frac{DY(s_o - s_1)}{D_c + k_d} \; . \qquad (27)$$

Defining

$$q = \frac{1}{X}\frac{dS}{dt} \qquad (28)$$

where: q = specific substrate metabolism rate

= mass of substrate consumed per unit mass of microorganisms per unit time, (T^{-1}).

From Equations 3 and 28 it can be shown that

$$q = \frac{\mu}{Y} . \tag{29}$$

Combining Equations 23 and 29 provides a fundamental relationship for continuous pure cultures growing on single substrates

$$D_c = Yq - k_d . \tag{30}$$

Equation 30 shows that the relationship of q and D_c is independent of the relationship between growth rate and growth-limiting substrate concentration. That is, the Monod equation (1) need not be valid for these expressions to be applicable. However, because of the relationship expressed in Equation 29 and the fact that μ must be functionally related to s_1, the residual growth-limiting substrate concentration, s_1, must be a direct function of the specific substrate metabolism rate, q.

COMPLEX ORGANISM—SUBSTRATE SYSTEMS

Kinetic Expression for Specific Growth Rate

This kinetic expression is derived using the assumption that the feed is composed of n individual constituents, w, and that at low concentrations each of these constituents is metabolized in a second-order reaction

$$\left(dw_i/dt\right)_{net} = - k'_{ij} x_j w_i \tag{31}$$

where k'_{ij} is the COD-based rate coefficient for microbiological species j consuming constituent i.

By materials balances it can be shown for a CSTR that

$$\frac{\left(w_i\right)_1}{\left(w_i\right)_o} = \frac{1}{1 + k'_{ij}\theta\left(x_j\right)_1} \tag{32}$$

where: $\left(w_i\right)_1$ = the concentration of constituent i in the reactor and effluent (as COD)

$\left(w_i\right)_o$ = the concentration of constituent i in the feed (as COD)

θ = hydraulic residence time = $\frac{1}{D}$.

If the composition of the feed is constant the COD of each feed constituent is proportional to the total COD of the feed

$$\left(w_i\right)_o = c_i z_o \tag{33}$$

where: z_o = total COD of feed

c_i = a constant.

Now, let

$$k'_{ij}\left(x_j\right)_1 = k_i x_1 \tag{34}$$

where x_1 = the total biomass concentration in the CSTR. Then, combining Equations 32, 33 and 34

$$\frac{\left(w_i\right)_1}{z_o} = \frac{c_i}{1 + k_i \theta x} \; . \tag{35}$$

However the total effluent COD is the sum of the COD's of the residual substrates

$$z_1 = \sum_{i=1}^{n} \left(w_i\right)_1 \tag{36}$$

where z_1 = residual COD of all substrates in the reactor and the effluent.

Therefore,

$$\frac{z_1}{z_o} = \sum_{i=1}^{n} \left(\frac{c_i}{1 + k_i \theta x_1}\right) . \tag{37}$$

Now, if there exists an overall rate coefficient, first order with respect to total substrate COD, then, using the same rationale as in the development of Equation 32, it can be shown that

$$\frac{z_1}{z_o} = \frac{1}{1 + k_o \theta x_1} \tag{38}$$

where k_o = the overall rate coefficient.

Such an overall rate coefficient can be found for any fixed operating condition, but there are only certain circumstances under which this rate coefficient will remain constant over a range of operating conditions. The following development shows that one of these conditions exists when nearly all of the feed constituents are more than about 80 percent metabolized. Combining Equations 37 and 38

$$\frac{1}{1 + k_o\theta x_1} = \sum_{i=1}^{n}\left(\frac{c_i}{1 + k_i\theta x_1}\right) . \qquad (39)$$

Expanding the LHS yields

$$\frac{1}{1 + k_o\theta x_1} = \sum_{p=1}^{\infty}\frac{(-1)^{p-1}}{\left(k_o\theta x_1\right)^p} \qquad (40)$$

Expanding the RHS yields

$$\sum_{i=1}^{n}\left(\frac{c_i}{1 + k_i\theta x_1}\right) = \sum_{i=1}^{n}\sum_{p=1}^{\infty}\frac{(-1)^{p-1}\,c_i}{\left(k_i\theta x_1\right)^p} . \qquad (41)$$

Inverting the summation procedure in Equation 41, one obtains

$$\sum_{i=1}^{n}\frac{c_i}{\left(k_i\theta x_1\right)} - \sum_{i=1}^{n}\frac{c_i}{\left(k_i\theta x_1\right)^2} + \sum_{i=1}^{n}\frac{c_i}{\left(k_i\theta x_1\right)^3} \cdots , \qquad (42a)$$

while Equation 40 yields

$$\frac{1}{k_o\theta x_1} - \frac{1}{\left(k_o\theta x_1\right)^2} + \frac{1}{\left(k_o\theta x_1\right)^3} \cdots . \qquad (42b)$$

The validity of Equation 39 therefore depends on the term by term equality of Equations 42a and 42b.

It is therefore concluded that, except for some special cases, k_o can only fulfill the constraints of Equations 42a and 42b and remain relatively constant if

$$\frac{1}{k_o\theta x_1} >> \frac{1}{\left(k_o\theta x_1\right)^2} \qquad (43)$$

and if

$$\sum_{i=1}^{n}\left(\frac{c_i}{k_i\theta x_1}\right) \gg \sum_{i=1}^{n}\frac{c_i}{\left(k_i\theta x_1\right)^2}\ , \tag{44}$$

which implies

$$k_i\theta x_1 \ll \left(k_i\theta x_1\right)^2\ . \tag{45}$$

The conditions of Equation 45 are fulfilled only when $(w_i)_1 \ll (w_i)_o$ and $z_1 \ll z_o$. For these conditions

$$\frac{1}{k_o} \simeq \sum_{i=1}^{n}\left(\frac{c_i}{k_i}\right)\ . \tag{46}$$

Rearranging Equation 38,

$$\frac{z_1}{z_o} = \frac{1}{k_o}\left[\frac{1}{\theta x_1}\left(1 - \frac{z_1}{z_o}\right)\right], \tag{47}$$

it can be seen that when k_o is constant, a plot of z_1/z_o and $1/\theta x_1$ $(1 - z_1/z_o)$ is linear with a slope of $1/k_o$. However, if there is a significant quantity of slowly degradable material in the feed, a plot of the form

$$\frac{z_1}{z_o} = \frac{1}{k_o'}\left[\frac{1}{\theta x_1}\left(1 - \frac{z_1}{z_o}\right)\right] + \frac{c_o}{k_o'} \tag{48}$$

will be obtained where:

k_o' = apparent overall rate coefficient

c_o = slowly degraded fraction of feed.

The behavior of a CSTR system into which two substrate constituents with different rate coefficients are fed serves to illustrate this kinetic expression:

Component 1	$c_a = 0.0555$	$k_a = 0.0526$ ℓ/mg-day
Component 2	$c_b = 0.9445$	$k_b = 19.0$ ℓ/mg-day .

A kinetic plot of the results predicted by Equation 48 is presented in Figure 3. Although Component 1 is only a small fraction of the feed, it causes significant nonlinearity in the lower region of the plot. It is only at higher values of the ordinate that k_o can be considered to be relatively constant, approaching the value of k_b. Projection of the limiting slope in Figure 3 yields an intercept on the abscissa of 0.0555, which is equivalent to c_a (equals c_o in Equation 48).

Based on this rationale it is postulated that the kinetic expression for specific growth rate can be characterized by an overall rate coefficient which is first order with respect to residual degradable organic matter concentration expressed as COD. The restricting conditions of this postulate are that inorganic nutrients are present in excess (a condition which is common in most waste treatment facilities), and that biologically resistant materials do not constitute more than a minor fraction of the feed COD. Under these conditions, the specific growth rate expression can be written as

$$\mu = k_o Y s_1$$

or

$$q = k_o s_1 \tag{49}$$

where: Y = yield of microorganisms per unit of substrate COD consumed

s_1 = concentration of degradable organic material expressed as COD, (ML^{-3})

k_o = overall rate coefficient for substrate removal, $(M^{-1}L^{3}T^{-1})$.

Cell Yield

Microbiological cell yield is defined as the quantity (mass or number) of live cells formed per unit quantity (mass) of growth-limiting substrate consumed. Since it is neither possible to measure the mass of living cell material nor the quantity of growth-limiting substrate in waste treatment systems, the determination of yield must be operationally defined and approximated.

Determination of total or volatile suspended solids (TSS or VSS) is the commonest method of assessing the amount of biological catalyst, but neither of these methods differentiate between active and inactive cell mass.

In an attempt to provide a more accurate representation of the active population in activated sludge, other more specific cell constituents such as DNA and cell protein have been used. However most of these techniques are quite tedious and there is little evidence that they provide a greater degree of certainty in their estimation of cell

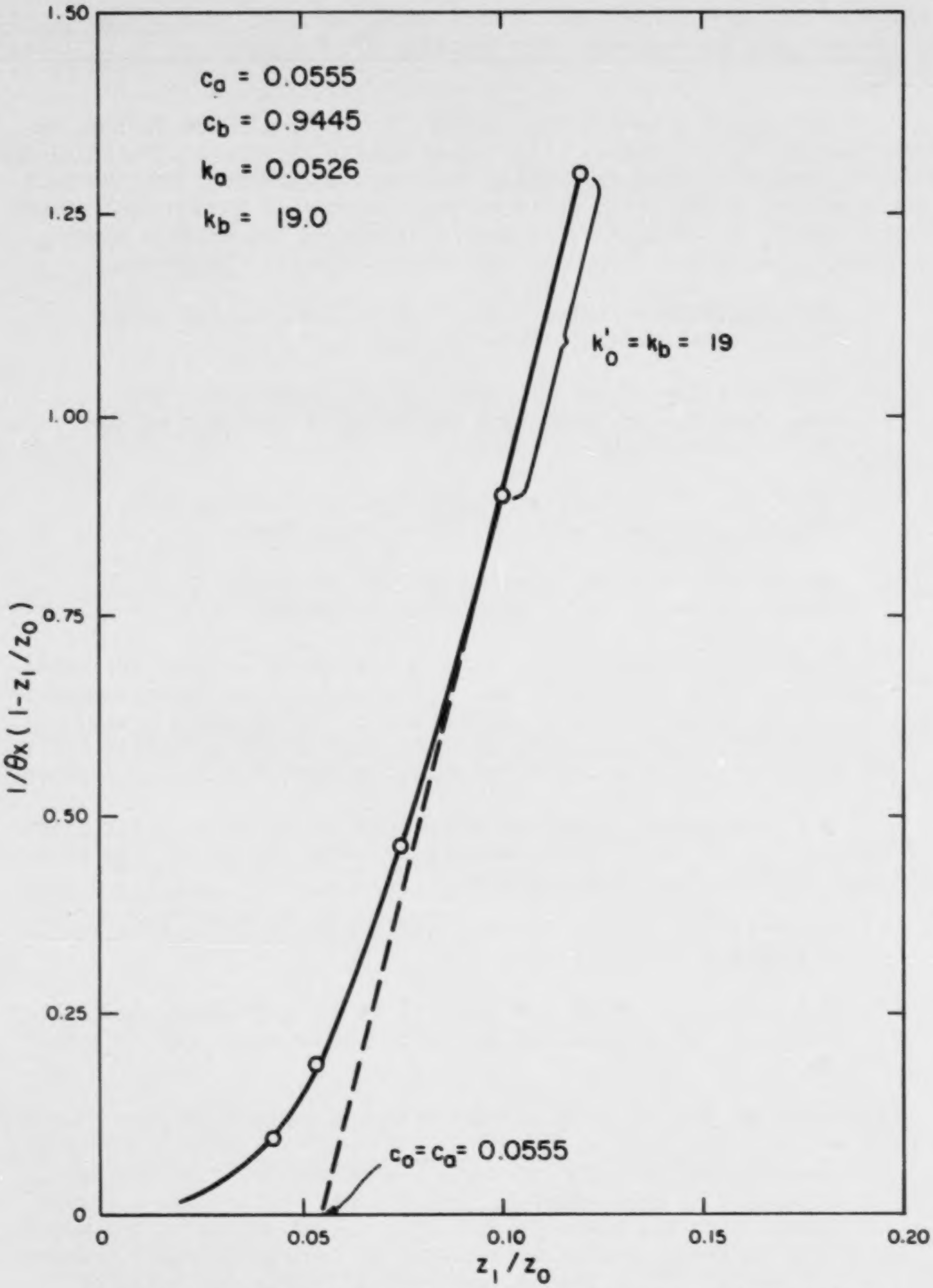


FIGURE 3. KINETIC PLOT FOR TWO-COMPONENT FEED

activity than VSS. Therefore in this study the term *volatile suspended solids was used to represent the quantity of cell material in activated sludge*.

The second quantity that appears in the yield coefficient is "growth-limiting substrate." In wastes treated by the activated sludge process, organic carbon is usually the limiting nutrient but the most common method of measuring waste strength is by the biochemical oxygen demand (BOD). Although this procedure estimates the aerobic growth potential of a waste, it has several disadvantages. These are:

1. The results are indirect since the consumption of oxygen by an unstandardized biota is observed.
2. The stoichiometry of the reactions is undefined. The test measures an incomplete and variable fraction of the biodegradable organic matter.
3. The 5-day time interval required to complete the test, besides being arbitrary, is undesirably long.
4. Oxygen depletion may result from the oxidation of noncarbonaceous constituents such as ammonia.

Organic carbon analysis, using a combustion [50] or wet oxidation technique [51] followed by CO_2 determination, can be performed rapidly and is an index of the organic matter content of a waste. However, the determination gives no indication of the oxidation state of the waste, a factor on which its growth potential strongly depends.

The measurement of COD by dichromate oxidation in boiling 50 percent H_2SO_4 [52] has distinct advantages as a measure of growth-limiting substrate in complex systems:

1. The oxidation is carried out with standardized reagents under standardized conditions.
2. The stoichiometry of the reaction is defined since the total oxidation requirement of the carbonaceous materials is determined.
3. The analysis can be performed within a reasonable period of time.
4. The difference in COD of waste before and after biological treatment is approximately proportional to the free energy change that has taken place during the oxidation of the waste. In a lumped parameter system such as biological waste treatment, the use of COD in lieu of free energy change is justified (Appendix A).

The COD test does have some disadvantages as a measure of growth-limiting substrate, primarily because it does not differentiate between biodegradable and nonbiodegradable organic matter. However, it is possible to develop a kinetic model and design experiments that will eliminate this factor.

Based on this rationale the following definition of yield was used in this investigation: The yield of biomass, expressed as VSS, is proportional to changes in substrate COD caused by the metabolism of the activated sludge microorganisms.

Endogenous Metabolism

The lack of knowledge concerning the true nature of endogenous metabolism has been discussed in Chapter II. It is also shown in Appendix A that cell decay and maintenance metabolism are difficult to differentiate under the typical experimental conditions used in biological waste treatment research.

Thus, because of the lack of more accurate information, it was assumed that the endogenous metabolism rate coefficient is a constant. For a completely mixed reactor the rate expression is:

$$V(dx/dt) = - Vk_d x \ . \qquad (50)$$

Residual Nondegradable COD

Since the COD analysis does not differentiate between degradable and nondegradable organic matter, the experimental program was designed to make such a differentiation possible. Because a soluble synthetic substrate consisting of readily degradable organic materials was used, it was assumed that the residual nondegradable materials were metabolic end products formed by the metabolism of the principal substrate.

While there are several kinetic expressions for the description of end product formation (Chapter II) for processes in which there are no forced changes in the physiological state of the microorganisms, there is only one kinetic description,

$$dP/dt = \alpha(dX/dt) - \beta(X) \qquad (19)$$

in which the fermentation constant, β, may be zero. It was assumed that Equation 19 (with $\beta = 0$) adequately described the production of metabolic end products in the activated sludge process. Subsequent data analysis indicated this assumption was reasonable.

For a CSTR system, the increase in biomass is given by the expression

$$dX/dt = V\mu x_1 \ .$$

In addition the growth rate, μ, can be defined by the substrate materials balance equation (25)

$$\mu = \frac{DY\left(s_0 - s_1\right)}{x_1} - \frac{Y}{x_1}\left(\frac{ds_1}{dt}\right) . \qquad (51)$$

Hence:

$$\frac{dX}{dt} = V\left[DY\left(s_0 - s_1\right) - Y\left(\frac{ds_1}{dt}\right)\right]. \qquad (52)$$

Having defined the rate of production of the nondegradable material (metabolic end products), a materials balance equation can be written

$$V\left(\frac{dp_1}{dt}\right) = 0 - Fp_1 + c_2V\left[DY\left(s_0 - s_1\right) - Y\left(\frac{ds_1}{dt}\right)\right] \qquad (53)$$

where: p_1 = concentration of end products in reactor, $\left(ML^{-3}\right)$

c_2 = constant of proportionality (α in Equation 19).

Defining

$$c_1 = c_2Y \qquad (54)$$

where c_1 = fraction of growth-limiting substrate converted to metabolic end products, then Equation 53 can be rearranged to the form of an ordinary differential equation

$$\frac{dp_1}{dt} + Dp_1 = c_1Ds_0 - c_1\left[Ds_1 + \left(\frac{ds_1}{dt}\right)\right]. \qquad (55)$$

INITIAL UPTAKE OF SUBSTRATE BY ACTIVATED SLUDGE

Several proposed mathematical models describing initial uptake of substrate were discussed in Chapter II. It will be the purpose of this section to identify and substantiate proposed mechanisms for initial uptake of soluble substrate and to develop a kinetic model based on the substantiated mechanism. Since laboratory experiments were confined to a soluble substrate, two mechanisms for initial uptake will be considered — adsorption and formation of intracellular storage products.

Adsorption

A conservative estimate of the magnitude of adsorptive initial uptake of substrate can be determined by examination of a hypothetical system in which a single small bacterium is immersed in a solution of an adsorbate with a large molecular diameter:

Consider a bacterium one micron (μ) in diameter in a solution containing spherical adsorbate molecules with a diameter of 100 Å (e.g., a spherical protein).

Bacterial surface area $= \pi\mu^3$.

Assuming close packing and a monolayer of adsorbate on the bacterial surface:

Volume of a 100-Å shell covering a 1-μ sphere
$= 10^{-2}\ \pi\mu^3$.

Volume of adsorbate molecules in this shell
$= \frac{\pi}{6}\ 10^{-2}\ \pi\mu^3$.

If bacterium and adsorbate have equal densities, then

$$\frac{\text{Volume of adsorbate}}{\text{Volume of bacterium}} = \frac{\pi/6\ 10^{-2}\ \pi\mu^3}{\pi/6\ \mu^3} = 0.0314\ .$$

Thus in such a hypothetical system a bacterium could adsorb 3.14 percent of its weight of substrate by initial uptake.

Since complete surface coverage is improbable, the figure of 0.0314 mg/mg probably represents a maximum for a single bacterial cell. In contrast to this hypothetical situation, activated sludge is usually made up of masses of organisms which agglomerate into flocs with dimensions of 50-100 μ or greater. In addition, most soluble substrate molecules are smaller than 100 Å in diameter and hence the maximum surface adsorption of soluble materials on activated sludge flocs is in all probability less than 0.0314 mg/mg. This conclusion is in agreement with Wuhrmann [36] and Heukelekian [26] who stated that adsorption is a minor mechanism in the initial uptake of soluble substrates.

Formation of Intracellular Storage Products

The formation of intracellular storage products, such as glycogen and polyhydroxybutyric acid (PHB), by microorganisms is well documented and has classically been associated with the metabolism of resting cells. In the absence of an essential growth factor a cell cannot multiply but some assimilative enzyme systems may continue to function deriving energy from a total oxidation of a portion of the substrate. Assimilation in the absence of multiplication results in the accumulation of storage products which can make up as much as 25 percent of the cell weight.

Storage product formation can also take place in the presence of surplus nutrients when cell division is unable to keep pace with assimilation as it does in balanced growth. Indeed, both glycogen and

PHB have been isolated from activated sludge organisms cultivated in batch systems where nutrients were in surplus [53,54].

Substantiation for the application of the storage product formation hypothesis to activated sludge systems has been obtained by reworking data from two literature sources: Gellman and Heukelekian [55] and Porges et al. [28]. Gellman and Heukelekian [55] carried out a series of batch experiments with several soluble industrial wastes, and performed simultaneous measurements of organic nutrient (BOD) uptake and oxygen consumption. The reworked data from these authors' experiments with the "yeast nutrients spent beer" waste are presented in Table VII and Figure 4. Sludge concentration is varied from 500 to 4000 mg/ℓ (TSS) and initial substrate concentration from 690 to 2760 mg/ℓ (BOD). While the absolute rates of oxygen uptake were dependent on both activated sludge and substrate concentrations, the relationship between oxygen consumption (percent of initial BOD) and substrate uptake (percent BOD removed) was virtually constant over much of the range of substrate assimilation (Figure 4). It would therefore appear that the functions of substrate uptake and oxygen consumption (substrate oxidation) are closely linked.

Porges et al. [28], in batch activated sludge experiments on a skim milk substrate used COD to measure substrate concentration and performed careful measurements of cell yield and oxygen utilization during balanced growth. Their data, presented in modified form in Table VIII and Figure 5, are in qualitative agreement with Gellman and Heukelekian [55] (Table VII and Figure 4). Figure 5 shows that the experimental ratio of oxygen consumption to substrate uptake is much lower than that predicted for balanced growth, indicating that substrate accumulation as intracellular storage products took place and that oxidation must occur subsequent to the initial uptake of substrate.

Kinetic Model

The kinetic model for initial uptake treated this reaction as an equilibrium sorption phenomenon in which the sorbed material consisted of small quantities of rapidly assimilated storage products and metabolic intermediates.

Although assimilation of substrate for the formation of storage products must be a rate phenomenon, it takes place with sufficient rapidity to warrant kinetic treatment as an equilibrium reaction. In addition, the experimental conditions used in this investigation – CSTR, moderately variable substrate concentration, and rapid cell recycle – did not favor substantial accumulation of storage products and were sufficiently restrictive to warrant the treatment of initial uptake in the above fashion.

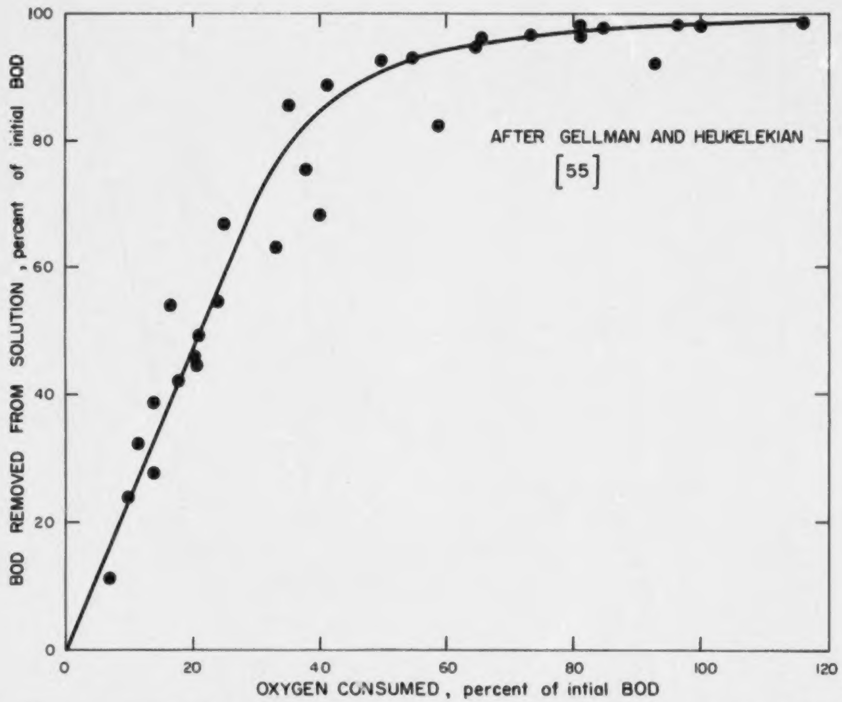


FIGURE 4. ASSIMILATION OF YEAST NUTRIENTS SPENT BEER BY ACTIVATED SLUDGE

TABLE VII

ASSIMILATION OF YEAST NUTRIENTS SPENT BEER BY ACTIVATED SLUDGE

(After Gellman and Heukelekian [55])

Sludge Solids mg/ℓ	Initial Waste BOD mg/ℓ		Time (hr)				
			2	4	6	12	24
500	1380	A[a]	10.9	27.5	45.6	88.3	96.0
		B[b]	6.9	14.1	20.3	41.3	65.5
1000	1380	A	23.9	42.0	54.3	92.6	97.7
		B	9.8	17.8	23.9	49.6	84.7
2000	690	A	45.0	68.1	81.9	92.0	98.5
		B	20.3	39.8	58.7	92.7	116.0
2000	1380	A	38.4	66.6	75.6	96.2	98.1
		B	13.8	25.0	37.7	73.1	96.2
2000	2760	A	31.9	49.3	63.1	94.6	97.9
		B	11.2	21.0	33.0	64.5	80.8
4000	1380	A	53.6	85.4	92.6	96.2	98.1
		B	16.3	35.1	54.7	81.1	100.0

[a] A = Percent of BOD removed from solution.

[b] B = Oxygen consumed – percent of initial BOD.

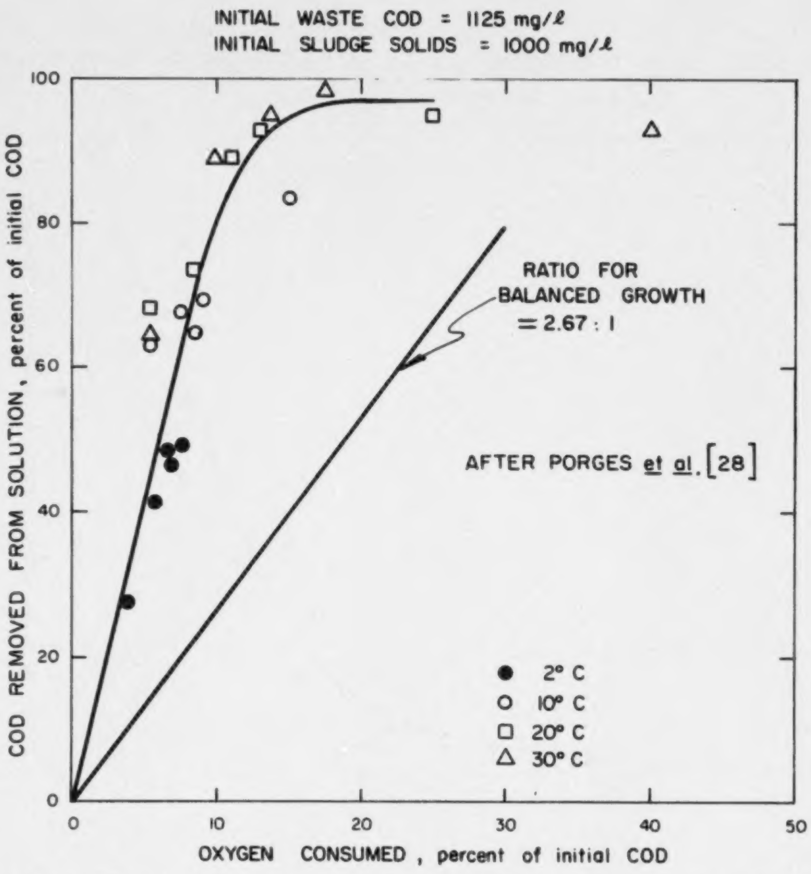


FIGURE 5. ASSIMILATION OF SKIM MILK BY ACTIVATED SLUDGE

TABLE VIII

ASSIMILATION OF SKIM MILK BY ACTIVATED SLUDGE

(After Porges, Jasewicz, and Hoover [28])

Temperature (°C)		Time (hr)				
		1	2	3	4	20
2	A[a]	27.6	41.3	48.3	46.4	49.1
	B[b]	3.9	5.7	6.6	6.8	7.1
10	A	63.1	67.6	64.6	69.2	83.5
	B	5.4	7.5	8.5	9.0	15.1
20	A	67.9	73.4	89.0	92.9	95.0
	B	5.3	8.3	11.0	13.1	24.9
30	A	64.0	89.2	94.6	98.3	92.4
	B	5.4	9.8	13.7	17.5	40.1

[a] A = Percent of initial COD removed from solution.

[b] B = Oxygen consumed — percent of initial COD.

Initial Waste COD - 1125 mg/ℓ

Initial Sludge Solids - 1000 mg/ℓ

It was assumed that the sorption expression was linear,

$$s_1 = \left(\frac{1}{\epsilon}\right)\frac{(s)_{ads}}{x_1} \tag{56}$$

where: s_1 = substrate concentration in reactor, (ML^{-3})

$(s)_{ads}$ = "sorbed" substrate concentration, (ML^{-3})

x_1 = sludge concentration in reactor, (ML^{-3})

ϵ = sorption coefficient, $(M^{-1}L^3)$.

Thus (for a constant sludge mass), the following term must be included in the substrate materials balance equation:

$$\left(\frac{ds}{dt}\right)_{ads} = \epsilon x_1\left(\frac{ds_1}{dt}\right) . \tag{57}$$

STEADY STATE CONDITIONS

A characteristic feature of a continuous process is its steady state operation. It is on this basis that comparisons of alternative process designs and evaluation of kinetic parameters are usually made.

The following development of steady state equations for the activated sludge process quantitatively accounts for metabolic end products and the removal of nutrients by "sorptive" mechanisms and allows the calculation of the overall substrate removal rate coefficient and the cell yield.

From Equations 21, 25, 49 and 57 materials balance equations can be derived for cell material (Equation 58) and growth-limiting substrate (Equation 59):

$$\frac{d(Vx_1)}{dt} = Fx_o - Fx_2 + k_oYs_1(Vx_1) - k_d(Vx_1) \tag{58}$$

$$\frac{d(Vs_1)}{dt} = Fs_o - Fs_2 - k_os_1(Vx_1) - \epsilon(Vx_1)\left(\frac{ds_1}{dt}\right) . \tag{59}$$

Growth-Limiting Substrate

At steady state, and with $x_o = 0$, Equation 58 becomes

$$\frac{Fx_2}{Vx_1} = k_oYs_1 - k_d = D_c . \tag{60}$$

Defining the steady state growth rate as

$$G = \left(D_c + k\right)_{\text{steady state}} = (\mu)_{\text{steady state}} \qquad (61)$$

the steady state growth-limiting substrate concentration can be written as

$$\left(s_1\right)_{\text{steady state}} = \frac{G}{k_o Y} \qquad (62)$$

which, when combined with Equation 29, can be written as follows

$$\left(s_1\right)_{\text{steady state}} = \left(\frac{1}{k_o}\right) q \; . \qquad (63)$$

When applied to most situations in biological waste treatment, Equation 63 contains two unknowns since besides the overall rate coefficient k_o, the concentration of growth-limiting substrate is unknown. In this investigation COD was used as a substrate parameter and its relationship to growth-limiting substrate was derived as follows.

Since a readily-degradable substrate was used in the laboratory, it is assumed that there was no undegradable material in the feed. Thus,

$$s_o = z_o \qquad (64)$$

where z_o = total COD of feed. The effluent soluble COD, z_1, will consist of residual substrate, s_1, and nondegradable metabolic end products, p_1, so that

$$z_1 = p_1 + s_1 \; . \qquad (65)$$

At steady state $dp_1/dt = 0$ and $ds_1/dt = 0$. Thus Equation 55 can be written

$$(p_1)_{\text{steady state}} = c_1 z_o - c_1 s_1 \; . \qquad (66)$$

Combining Equations 64, 65, and 66 and solving for z_1 we obtain

$$z_1 = c_1 z_o + (1 - c_1) s_1 \; . \qquad (67)$$

Substituting for s_1 from Equation 63 yields

$$z_1 = c_1 z_o + \frac{(1 - c_1)}{k_o} q \; . \qquad (68)$$

The specific substrate removal rate based on COD is defined as

$$q_z = \frac{D(z_o - z_1)}{x_1} . \qquad (69)$$

Therefore

$$\frac{q}{q_z} = \frac{(s_o - s_1)}{(z_o - z_1)} . \qquad (70)$$

Combining Equations 67 and 70, it can be shown that

$$q\left\{z_o - \left[c_1 z_o + \left(1 - c_1\right) s_1\right]\right\} = q_z\left(z_o - s_1\right) \qquad (71)$$

which reduces to

$$q_z = q\left(1 - c_1\right) . \qquad (72)$$

Equation 68 can now be written

$$z_1 = c_1 z_o + \left(\frac{1}{k_o}\right) q_z \qquad (73)$$

which on division by z_o yields a two-dimensional linear form

$$\frac{q_z}{z_o} = k_o \left(\frac{z_1}{z_o}\right) - c_1 k_o . \qquad (74)$$

Data plotted in the form of Equation 74 can be statistically analyzed to obtain the overall rate coefficient, k_o, from the slope and the limiting residual COD, c_1, from the abscissa intercept. Having determined (c_1 and k_o) the concentration of rate-limiting substrate can be calculated from Equation 67 or from a combination of Equations 63 and 64.

Cell Material

At steady state for a CSTR ($s_2 = s_1$), Equation 59 becomes

$$\left(x_1\right)_{\text{steady state}} = \frac{F(s_o - s_1)}{V k_o s_1} = \frac{D(s_o - s_1)}{k_o s_1} . \qquad (75)$$

Combining Equations 75 and 63 and substituting for s_1 we obtain

$$\left(x_1\right)_{\text{steady state}} = DY\left[\frac{s_o}{G} - \frac{1}{k_oY}\right]. \tag{76}$$

Now from Equation 30, it can be seen that

$$Yq = \left(D_c + k_d\right) = \text{constant} .$$

Combining Equations 30 and 72, the relationship between the COD-based yield, Y_z, and the yield based on growth-limiting substrate, Y, can be determined as

$$Y = \left(1 - c_1\right)Y_z . \tag{77}$$

Substituting Equation 77 into Equation 76 we obtain

$$\left(x_1\right)_{\text{steady state}} = D\left[\frac{Y_z\left(1 - c_1\right)z_o}{G} - \frac{1}{k_o}\right]. \tag{78}$$

The values of c_1 and k_o are determined from a steady state plot of Equation 74, and Y_z is determined by a plot of steady state data in the form of Equation 30.

Summary

The steady state equations developed above allow the experimental evaluation of the overall rate coefficient, k_o, the limiting soluble COD fractional residual, c_1, the growth-limiting substrate concentration, the COD-based yield, Y_z, the endogenous metabolism rate coefficient, k_d, and the biomass concentration at various operating conditions.

UNSTEADY STATE

Introduction

Three types of transient input conditions were studied in laboratory experiments: 1) step transients; 2) pulse transients (δ-function); and 3) periodic transients (diurnal). The theoretical response of the activated sludge systems to each of these conditions is treated individually.

In general the responses of biological processes to transients can be classified as "short term" and "long term" because the two time factors D and G usually differ by at least an order of magnitude.

Short-term transients typically occur in time periods of about one hour whereas long-term changes take place over one or more days. In the developments that follow, certain assumptions are made based on the difference in the time factors D and G so that the simultaneous solution of the materials balance equations 21 and 25 is possible. The assumptions made to permit these solutions are indicated in the relevant sections.

Step Transients

Short Time Periods: Growth-Limiting Substrate.

The assumption is made that $dx_1/dt = 0$ and therefore

$$x_1 = \text{constant} = \bar{x}_1 \ . \qquad (79)$$

Using this assumption and setting $s_2 = s_1$ for a CSTR, Equation 59 becomes

$$\frac{ds_1}{dt} = Ds_o - \left(k_o\bar{x}_1 + D\right)s_1 - \epsilon\bar{x}_1\left(\frac{ds_1}{dt}\right) . \qquad (80)$$

The short-term solution for substrate concentration is obtained by integrating Equation 80,

$$s_1 = \frac{Ds_o}{k_o\bar{x}_1 + D} - \left[\frac{Ds_o}{k_o\bar{x}_1 + D} - \left(s_1\right)_o\right]e^{-\left(\frac{k_o\bar{x}_1+D}{1+\epsilon\bar{x}_1}\right)t} \qquad (81)$$

where $(s_1)_o = s_1$ at $t = 0$. The asymptotic value of s_1 indicated by Equation 81 is

$$\langle s_1\rangle = \frac{Ds_o}{k_o\bar{x}_1 + D} \ . \qquad (82)$$

By regrouping the variables Equation 81 can be linearized thus:

$$\ln\left[\frac{s_1 \quad - \langle s_1\rangle}{(s_1)_o - \langle s_1\rangle}\right] = \left[\frac{k_o\bar{x}_1 + D}{1 + \epsilon\bar{x}_1}\right]t \ . \qquad (83)$$

Short Time Periods: Total Soluble COD.

For simplification let

$$\left[\frac{k_o\bar{x}_1 + D}{1 + \epsilon\bar{x}_1}\right] = \lambda \qquad (84)$$

and

$$<s_1> - \left(s_1\right)_o = \Delta s_1 \ . \qquad (85)$$

Then differentiating Equation 81 with respect to time gives

$$\frac{ds_1}{dt} = \lambda \left(\Delta s_1\right) e^{-\lambda t} \ . \qquad (86)$$

Substituting Equation 82 and 86 into Equation 55 yields

$$\begin{aligned} \frac{dp_1}{dt} + Dp_1 &= c_1 D z_o + c_1 \left[\left(D - \lambda\right)\left(\Delta s_1\right) e^{-\lambda t} - D <s_1>\right] \\ &= c_1 D \left(z_o - <s_1>\right) + c_1 \left(D - \lambda\right)\left(\Delta s_1\right) e^{-\lambda t} \ . \end{aligned} \qquad (87)$$

Integration of Equation 87 gives

$$p_1 = c_1\left(z_o - <s_1>\right)\left(1 - e^{-Dt}\right) + c_1\left(\Delta s_1\right)\left(e^{-\lambda t} - e^{-Dt}\right) + \left(p_1\right)_o e^{-Dt} \ . \qquad (88)$$

Now the total soluble COD, z_1, is the sum of Equations 81 and 88 which can be simplified, because in most cases

$$\lambda \gg D,$$

$$e^{-\lambda t} \to 0, \text{ and}$$

$$\left(\Delta s_1\right) \to 0 \ .$$

In such cases an approximate expression for z_1 is

$$z_1 = c_1 z_o + <s_1>\left(1 - c_1\right) - \left[c_1\left(z_o - <s_1>\right) - \left(p_1\right)_o\right] e^{-Dt} \ . \qquad (89)$$

Taking a limit as t becomes large results in the expression

$$\frac{<q_z>}{z_o} = k_o \left[\frac{<z_1>}{z_o}\right] - c_1 k_o \qquad (90)$$

where $<z_1>$ = the short time period asymptotic value of z_1, and

$$<q_z> = \frac{D\left(z_o - <z_1>\right)}{x_1} \ . \qquad (91)$$

Thus the short time asymptotic soluble COD values obtained from step transient input experiments should plot on the same operating line as the steady state results given by Equation 74.

Long-Term Changes: Biomass. When the substitution $G = D_c + k_d$ is made, Equation 58 can be written

$$\frac{dx_1}{dt} = \left(k_o Y s_1 - G\right) x_1 \ . \qquad (92)$$

To eliminate the parametric function, s_1, Equation 80 can be rearranged to give

$$s_1 = \frac{Ds_o}{k_o x_1 + D} - \left[\frac{1 + \epsilon x_1}{k_o x_1 + D}\right] \frac{ds_1}{dt} \ . \qquad (93)$$

Now usually for long-term changes

$$Ds_o > 1000 \text{ mg}/\ell\text{-day},$$

$$1 + \epsilon x_1 < 2, \text{ and}$$

$$ds_1/dt < 10 \text{ mg}/\ell\text{-day}.$$

Thus,

$$ds_1/dt \ll \frac{Ds_o}{1 + \epsilon x_1}$$

and therefore

$$s_1 \simeq \frac{Ds_o}{k_o x_1 + D} \ . \qquad (94)$$

The term $k_o x_1$ in Equation 94 is usually at least one order of magnitude greater than D, ($k_o x_1 > 100 \text{ day}^{-1}$, $D < 10 \text{ day}^{-1}$) and therefore D can be ignored in the denominator term. With these approximations Equation 92 becomes

$$\frac{dx_1}{dt} + Gx_1 = DYs_o$$

which on integration gives

$$x_1 = \frac{DYs_o}{G} - \left[\frac{DYs_o}{G} - \left(x_1\right)_o\right] e^{-Gt} \ . \qquad (95)$$

From Equation 76, the ultimate biomass concentration $\left(x_1\right)_\infty$ is

$$\left(x_1\right)_\infty = DY\left[\frac{s_o}{G} - \frac{1}{k_oY}\right] .$$

Hence Equation 95 can be modified to

$$x_1 = \left(x_1\right)_\infty + \frac{D}{k_o} + \left[\left(x_1\right)_o - \left(x_1\right)_\infty - \frac{D}{k_o}\right]e^{-Gt} . \tag{96}$$

Grouping the variables we obtain

$$\ln\left\{\frac{\left(x_1\right)_o - \left[\left(x_1\right)_\infty + \frac{D}{k_o}\right]}{x_1 - \left[\left(x_1\right)_\infty + \frac{D}{k_o}\right]}\right\} = Gt. \tag{97}$$

In general

$$x_1 \leqq 1000 \text{ mg}/\ell, \text{ and}$$

$$\frac{D}{k_o} \leqq \frac{10}{0.25} = 40 .$$

Thus

$$x_1 \gg \frac{D}{k_o}$$

so that a plot of the function $\ln\left[x_1 - \left(x_1\right)_\infty\right]$ and t should be linear with a slope equal to -G. This plot provides a technique for determining the variation of the endogenous metabolism coefficient, k_d, as a function of growth rate, since $G = D_c + k_d$, and the value of D_c can be readily determined by experiment.

Long-Term Changes: Growth-Limiting Substrate. If no change in the cell dilution rate is embodied in the step transient, then s_1 must return to its initial steady state value after being displaced to the asymptotic value, $\langle s_1\rangle$, described by Equation 82. This long-term change is described by Equation 94, which when inverted becomes

$$\frac{1}{s_1} = \frac{1}{s_o} + \left(\frac{k_o}{Ds_o}\right)x_1 . \tag{98}$$

Substituting for x_1 from Equation 96 results in the expression

$$\frac{1}{s_1} = \frac{1}{s_o} + \frac{k_o(x_1)_\infty + D}{Ds_o} + \left[\frac{k_o(x_1)_o}{Ds_o} - \frac{k_o(x_1)_\infty + D}{Ds_o}\right]e^{-Gt} \ . \tag{99}$$

The ultimate, or steady state, value for s_1 can be written,

$$\frac{k_o(x_1)_\infty + D}{Ds_o} = \frac{1}{(s_1)_\infty} \tag{100}$$

and

$$\frac{k_o(x_1)_o}{Ds_o} = \frac{1}{\langle s_1\rangle} - \frac{1}{s_o} \ . \tag{101}$$

Hence Equation 99 can be written

$$\frac{1}{s_1} = \frac{1}{s_o} + \frac{1}{(s_1)_\infty} + \left[\frac{1}{\langle s_1\rangle} - \frac{1}{s_o} - \frac{1}{(s_1)_\infty}\right]e^{-Gt} \ , \tag{102}$$

or grouping the variables gives

$$\ln\left\{\frac{\frac{1}{\langle s_1\rangle} - \left[\frac{1}{s_o} + \frac{1}{(s_1)_\infty}\right]}{\frac{1}{s_1} - \left[\frac{1}{s_o} + \frac{1}{(s_1)_\infty}\right]}\right\} = Gt \ . \tag{103}$$

Long-Term Changes: Total Soluble COD. Because it is assumed that the concentration of end products is proportional to the term $(s_o - s_1)$ and $s_o \gg s_1$, long-term variations in end product concentration should be extremely small. Thus for most cases of interest the long-term end product concentration is approximately a constant

$$(p_1)_{\text{long term}} \simeq c_1\left[z_o - (s_1)_\infty\right] . \tag{104}$$

Since

$$z_1 = p_1 + s_1 \tag{65}$$

and from Equation 102

$$s_1 = \left\{\frac{1}{s_o} + \frac{1}{(s_1)_\infty} + \left[\frac{1}{\langle s_1\rangle} - \frac{1}{s_o} - \frac{1}{(s_1)_\infty}\right]e^{-Gt}\right\}^{-1} , \tag{105}$$

z_1 is approximately the sum of Equations 104 and 105.

Pulse Transients (δ-function)

Because the experimental use of pulse inputs was restricted to the study of initial substrate removal, only short-term behavior was of interest. The short-term time dependent behavior for continuous flow reactors subjected to pulse inputs can be determined from Equation 81 by letting

$$\left(s_1\right)_o = \left(s_1\right)_{o+} = \frac{S}{V(\epsilon x_1)} + \left(s_1\right)_{o-} \qquad (106)$$

where: S = mass of substrate in pulse

$\left(s_1\right)_{o+}$ = growth-limiting substrate concentration in system immediately after adding the pulse

$\left(s_1\right)_{o-}$ = growth-limiting substrate concentration in system prior to adding the pulse.

Only limited use was made of pulse transients in continuous flow apparatus, because the initial uptake phenomenon which they were designed to detect was confounded by dispersion in the cell separator.

Periodic Transients

Instrumental difficulties prevented exact reproduction of the periodic transients typically encountered in domestic waste flow and strength. A sawtooth function was used to approximate these variations. For such a function the development of solutions for total soluble COD residual, z_1, requires that the rate-limiting substrate, s_1, and the end products, p_1, be considered separately and then added. Several simplifying assumptions are made to avoid mathematical unwieldiness in the following expressions. Rearranging Equation 80, the differential equation for s_1 is obtained in the standard form,

$$\frac{ds_1}{dt} + \left[\frac{k_o x_1 + D}{1 + \epsilon x_1}\right] s_1 = \frac{D s_o}{1 + \epsilon x_1} \, . \qquad (107)$$

As in previous developments, the following assumptions are made:

$s_o = z_o$ = the feed COD.

$x_1 = \bar{x}_1$ = the average biomass concentration.

Additionally it is assumed that

$D = \bar{D}$ = the average hydraulic dilution rate.

Because of the magnitude of the term $k_o x_1$ (Equation 107), D is insignificant in most practical situations. For notational simplicity the following definitions are made:

$$\left[\frac{k_o\bar{x}_1 + \bar{D}}{1 + \epsilon\bar{x}_1}\right] = \lambda \ , \tag{108}$$

$$\left[\frac{Dz_o}{1 + \epsilon\bar{x}_1}\right]_{min} = \lambda_5 \ , \text{ and} \tag{109}$$

$$\left[\frac{Dz_o}{1 + \epsilon\bar{x}_1}\right]_{max} = \lambda_5 + \lambda_6 \ . \tag{110}$$

The basic differential equation now becomes

$$\frac{ds_1}{dt} + \lambda s_1 = \lambda_5 + \frac{\lambda_6}{c}\left[H(c,t)\right] \tag{111}$$

where $H(c,t)$ = the sawtooth function with a period of 2c. Considering the terms on the right of Equation 111 individually, the first contributes the following to the long-term periodic solution:

$$\lim_{t \to \infty}\left(s_1, \text{ term } 1\right) = \frac{\lambda_5}{\lambda} \ . \tag{112}$$

The contribution of the second term, $(\lambda_6/c)[H(c,t)]$, depends on T, the time from the beginning of the period. Due to the length of the mathematical derivation, it is presented in Appendix B, and only the solution is included below. Because the function $H(c,t)$ has a discontinuous first derivative, the solution is discontinuous and in two parts, which are

(i) For $0 \leq T \leq c$

$$\lim_{t \to \infty} s_1 = \frac{\lambda_5}{\lambda} + \frac{\lambda_6}{\lambda^2 c}\left\{\left[1 + \tanh\left(\frac{\lambda c}{2}\right)\right]e^{-\lambda T} + \lambda T - 1\right\} ; \tag{113a}$$

(ii) For $c \leq T \leq 2c$

$$\lim_{t \to \infty} s_1 = \frac{\lambda_5}{\lambda} + \frac{\lambda_6}{\lambda^2 c}\left\{\left[1 + \tanh\left(\frac{\lambda c}{2}\right)\right]e^{-\lambda T} - 2e^{-\lambda(T-c)} + \lambda(2c - T) + 1\right\} \tag{113b}$$

where T = time referenced to the minimum point of the sawtooth function, i.e., the beginning of the period. T is always less than 2c, the length of the period.

End product variations are described by Equation 53 — but for cases in which D is a periodic function, the equation is nonlinear and a generalized solution is extremely complex. Therefore, with one exception, only linear cases have been considered in this study and it is assumed that

$$\frac{dp_1}{dt} + Dp_1 \cong c_1 Dz_o \ . \tag{114}$$

The two periodic conditions investigated were: 1) feed strength variations at constant flow rate; and 2) flow variations at constant feed strength. For the first case, the solution is very similar to Equation 113. Setting

$$\left(c_1 Dz_o\right)_{min} = \lambda_7 \tag{115}$$

and

$$\left(c_1 Dz_o\right)_{max} = \lambda_7 + \lambda_8 \ , \tag{116}$$

the following solutions are obtained:

(i) For $0 \leqq T \leqq c$

$$\lim_{t \to \infty} p_1 = \frac{\lambda_7}{D} + \frac{\lambda_8}{D^2 c}\left\{\left[1 + \tanh\left(\frac{Dc}{2}\right)\right] e^{-DT} + DT - 1\right\} ; \tag{117a}$$

(ii) For $c \leqq T \leqq 2c$

$$\lim_{t \to \infty} p_1 = \frac{\lambda_7}{D} + \frac{\lambda_8}{D^2 c}\left\{\left[1 + \tanh\left(\frac{Dc}{2}\right)\right] e^{-DT} - 2e^{-D(D-c)} + D(2c - T) + 1\right\} . \tag{117b}$$

When only the flow rate is varied (D is a periodic variable) and the feed strength is kept constant, the expression for end products is nonlinear. Within the limits of the validity of Equation 114, that is if s_1 and ds_1/dt do not significantly affect p_1, then p_1 is a constant if z_o is not varied, regardless of the functional nature of D, and

$$p_1 \simeq c_1 z_o \ . \tag{118}$$

The response of total soluble COD is calculated by adding Equations 113a and 113b to Equations 117a and 117b or to Equation 118, depending upon the nature of the periodic input.

IV. EXPERIMENTAL PHASE

CONTINUOUS FLOW LABORATORY EXPERIMENTS

Construction and Operation of Apparatus

Completely mixed (CSTR) reactors, consisting of Griffen beakers in which the liquid volume was maintained at either 800 mℓ or 1600 mℓ, were used for the continuous flow experiments. The reactor contents were mixed with 2-inch x 3-inch stirring paddles driven at 60 rpm by synchronous motors (Cramer Model 117). Laboratory compressed air was introduced to the reactors at a rate of 0.2 to 0.4 cfh through porous stone diffusers after passing through an oil trap, water scrubbers, and rotameters (Brooks-Mite Model 1-38B). These relatively high air rates were essential for adequate mixing.

Mixed liquor was siphoned from the reactors into upflow clarifiers. The 800-mℓ reactors had clarifiers constructed from 40-mℓ centrifuge tubes whose effective volume was 36 mℓ, and approximately 20 mℓ of which was above the point of siphon discharge. For the 1600-mℓ reactors, clarifiers were constructed of 1-1/2 inch diameter glass tubing, drawn to a conical shape at the bottom. The upflow zone of these clarifiers was about 40 mℓ and they had a total volume of 100 mℓ. The nominal overflow rate of these clarifiers was approximately 125 gpd /sq ft under standard operating conditions, and was increased as much as fourfold under certain transient flow conditions. A relatively high recycle ratio of 4:1 was maintained throughout the experiments to minimize the departure from completely mixed conditions caused by the clarifiers.

All pumping was performed with peristaltic pumps using Tygon tubing. The feed and cell recycle lines passed through the same pump and the recycle ratio was established by choice of tubing size. Feed-recycle pumps were the Sigmamotor Model T-6 driven by variable speed DC motors (Bodine Model NSH-54RL motors with Minarik Model SH-53 controllers). The smaller Sigmamotor Model T-8 pumps were used for cell wasting. These pumps were driven by AC motors through variable speed transmissions (Zeromax) and were operated on an intermittent basis by interval timers (Western Electro-Mechanical Co.).

Feed containers were 5-gal glass carboys which normally contained sufficient feed for one week of operation. The units were located in cabinets maintained at 20° ± 1° C.

A pictorial diagram of a typical reactor system is shown in Figure 6; Figure 7 illustrates the mode of operation; and Figure 8 is a photograph of a reactor with its related equipment. The feed material was pumped from the reservoir, A, through the combined

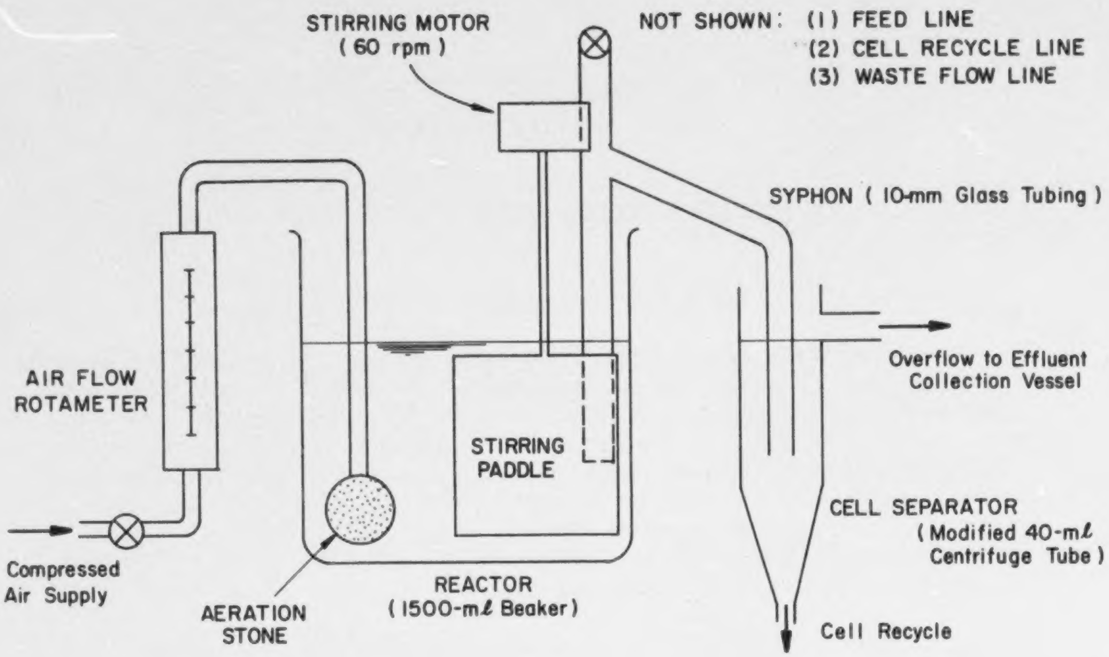

STIRRING MOTOR
(60 rpm)
NOT SHOWN: (1) FEED LINE
(2) CELL RECYCLE LINE
(3) WASTE FLOW LINE
SYPHON (10-mm Glass Tubing)
AIR FLOW
ROTAMETER
Overflow to Effluent
Collection Vessel
STIRRING
PADDLE
CELL SEPARATOR
(Modified 40-mℓ
Centrifuge Tube)
Compressed
Air Supply
AERATION
STONE
REACTOR
(1500-mℓ Beaker)
Cell Recycle

FIGURE 6 . PICTORIAL DIAGRAM OF LABORATORY ACTIVATED SLUDGE UNIT

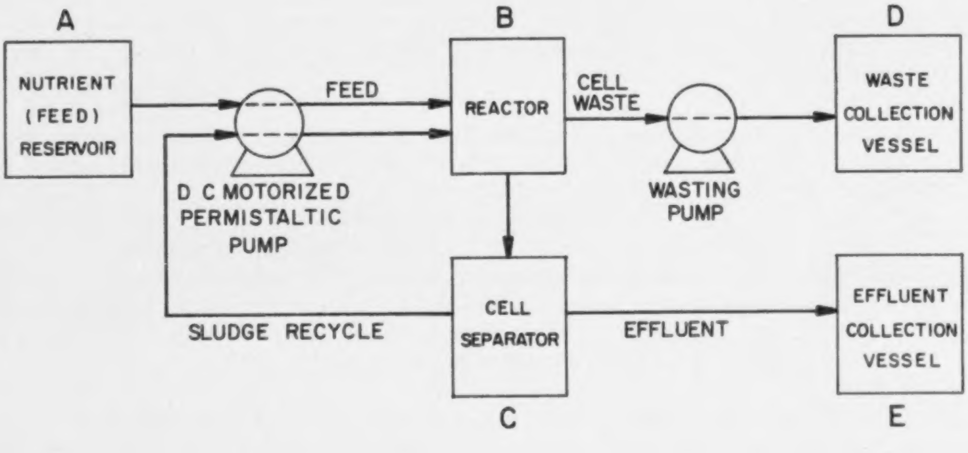

A
NUTRIENT
(FEED)
RESERVOIR
FEED
B
REACTOR
CELL
WASTE
D
WASTE
COLLECTION
VESSEL
D C MOTORIZED
PERMISTALTIC
PUMP
WASTING
PUMP
CELL
SEPARATOR
SLUDGE RECYCLE
EFFLUENT
EFFLUENT
COLLECTION
VESSEL
C
E

FIGURE 7. BLOCK FLOW DIAGRAM OF LABORATORY ACTIVATED SLUDGE UNIT

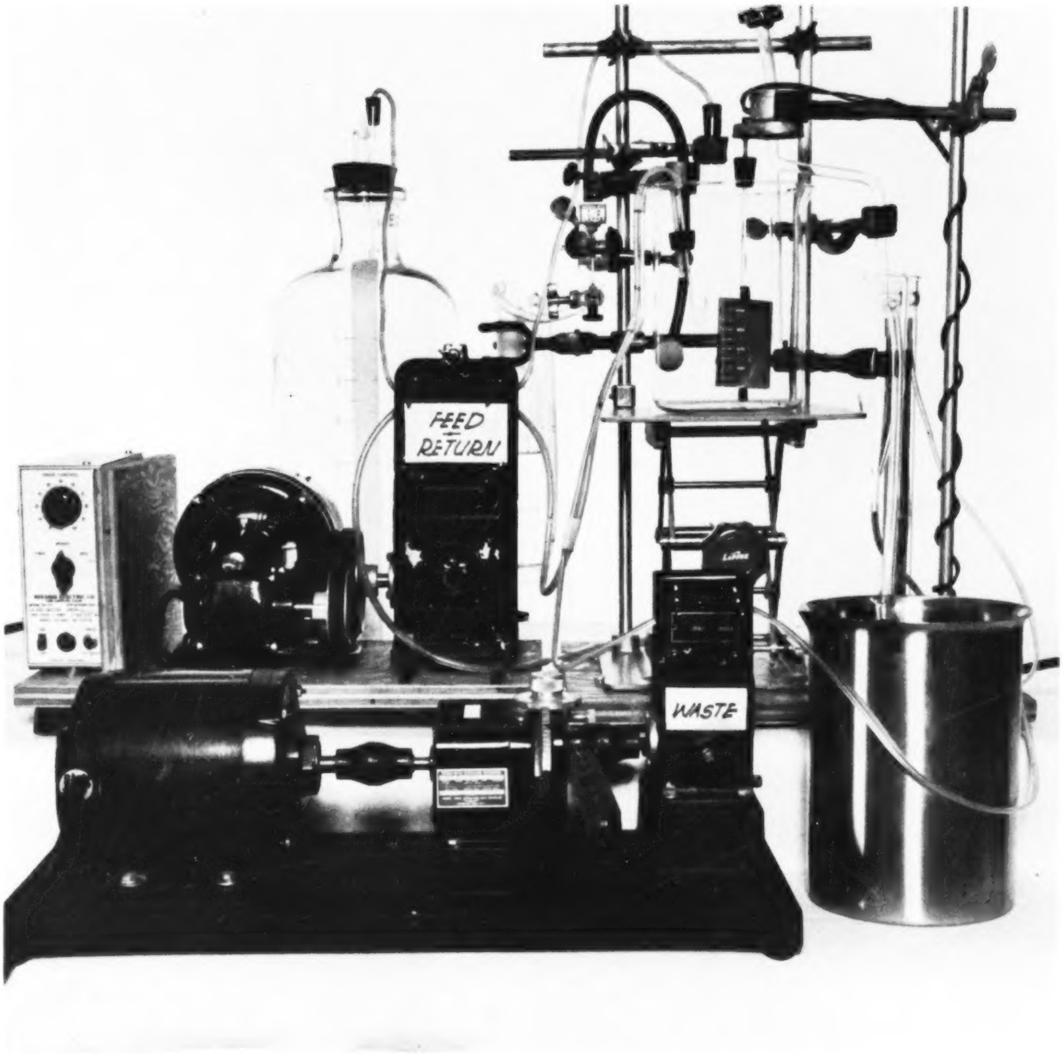

FEED
RETURN
WASTE

FIGURE 8. LABORATORY ACTIVATED SLUDGE UNIT

feed-recycle pump into the reactor, B. The mixed reactor contents then siphoned into the clarifier, C, from which settled cell material was returned to the reactor. The clarifier overflow discharged into the effluent collection vessel, E. Cells were wasted directly from the reactor since this provided an accurate control of wasting rates. The wasted cells were pumped to the waste collection vessel, D.

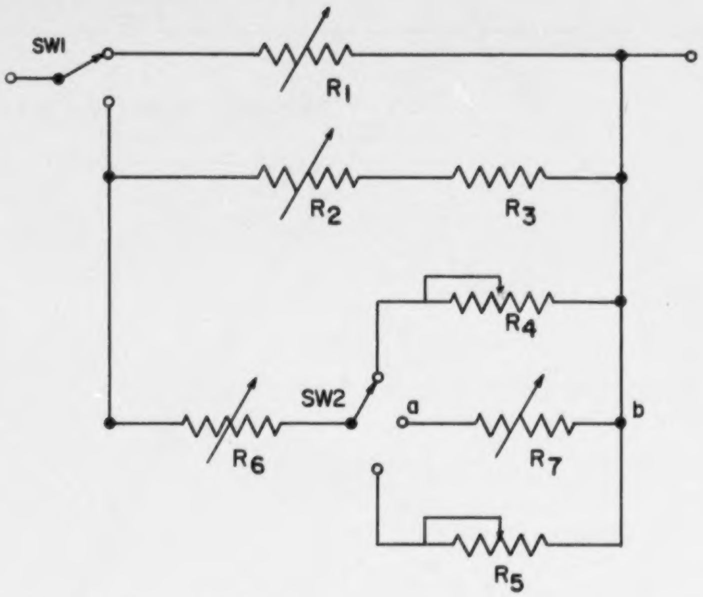
Flow rates were established from daily readings of the volumes of liquid in the feed reservoirs, effluent collection vessels, and waste collection vessels. Incremental volumes were divided by the elapsed time to obtain rates of flow.

Step transient inputs were effected by a manual instantaneous change of the feed rate, the feed strength, or the rate of sludge wasting.

Periodic transients were obtained by programming the feed pumps with sawtooth function controllers which (Figures 9 and 10) were substituted for the speed control potentiometer in the Minarik SH-53 speed controllers. The sawtooth controllers consisted of a circuit of resistors, the overall resistance of which varied with time in a sawtooth pattern. The minimum and maximum resistances of the sawtooth controller, R_2 and R_6, fixed the minimum and maximum feed pumping rates and were adjusted to the desired values at the start of each experiment. During an experiment the resistance of the programmed speed control, R_7, was varied in a sawtooth pattern and this pattern was reflected in the pumping rate. The programmed speed control (Figure 10) consisted of two wirewound potentiometers and a microswitch cam, both operated by a common rotating shaft driven by a synchronous motor (Cramer) at a speed of one revolution per day. The sweepers for the wirewound potentiometers were fixed on the shaft 180° out of phase. The microswitch cam was adjusted so that each potentiometer functioned only over the central portion of its resistance (2500 to 7500 Ω). Thus after the resistance of the programmed speed control circuit had traversed the range of 2500 Ω to 7500 Ω on one of the wirewound potentiometers, the microswitch was tripped and the resistance decreased to 2500 Ω as the sweeper moved across the second wirewound potentiometer. At this point the microswitch was tripped back to its original position, and the process was repeated. Figure 11 is a plot of the entire circuit resistance as a function of cycle time for an experiment in which the overall resistance range was 3000 Ω. By using two identical sawtooth controllers, operated 180° out of phase, on substrates of different strength the feed flow rate could be maintained at a constant level while the feed strength was varied.

Substrate

A substrate for laboratory experiments on biological waste treatment should fulfill several criteria, the most important of which is that it should be representative of the constituents typically found in the waste flows being studied. It should also be reproducible and there should be a relatively easy method of separating the substrate from cell material.


SW1
R1
R2
R3
R4
SW2
a
b
R6
R7
R5

SW1	SPDT Toggle Switch
SW2	3-Position Rotary Switch
R_1	Normal Speed Control, 6.8K Ω
R_2	Minimum Speed Adjustment, 20K Ω - 5W
R_3	5K Ω - 5W
R_4	Maximum Speed Shunt Resistor, 3K Ω - 5W (Resistance fixed at time of construction.)
R_5	Minimum Speed Shunt Resistor, 10K Ω - 5W (Resistance fixed at time of construction.)
R_6	Maximum Speed Adjustment
R_7	Programmed Speed Control (See detail.)

SAWTOOTH FUNCTION SPEED CONTROL FOR MINARIK SH-53 CONTROL UNIT OPERATING BODINE NSH-54RL DC MOTOR

FIGURE 9. SAWTOOTH FUNCTION SPEED CONTROL

R_{7a}, R_{7b}	10K Ω - 50 W Wirewound Potentiometers (Linear Taper), Modified for 360° Rotation
SW3	Cam-actuated SPDT Microswitch

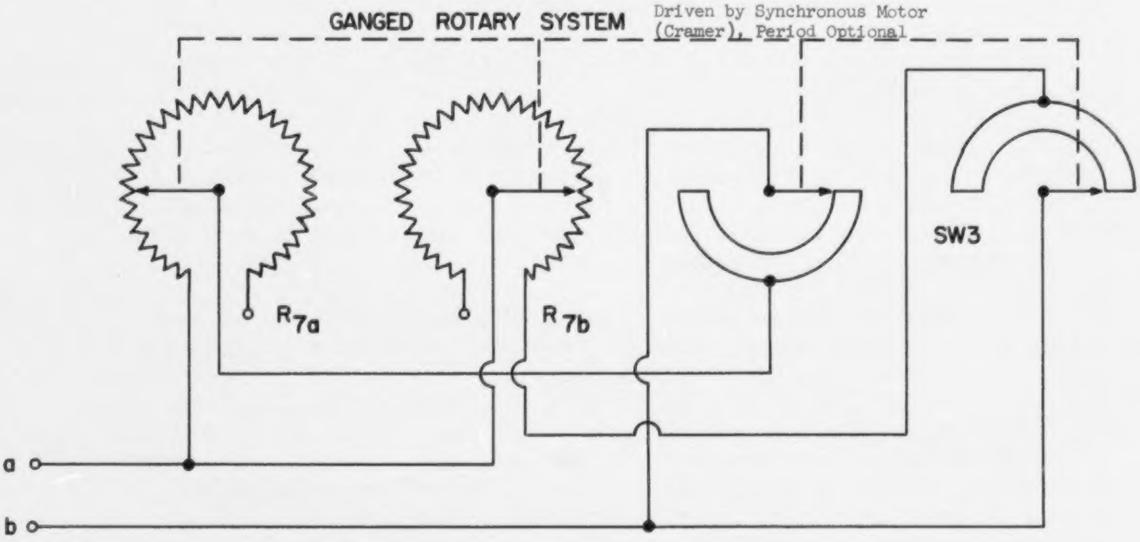

GANGED ROTARY SYSTEM
Driven by Synchronous Motor (Cramer), Period Optional
R7a
R7b
SW3
a
b

FIGURE 10. DETAIL OF PROGRAMMED SPEED CONTROL FOR SAWTOOTH FUNCTION

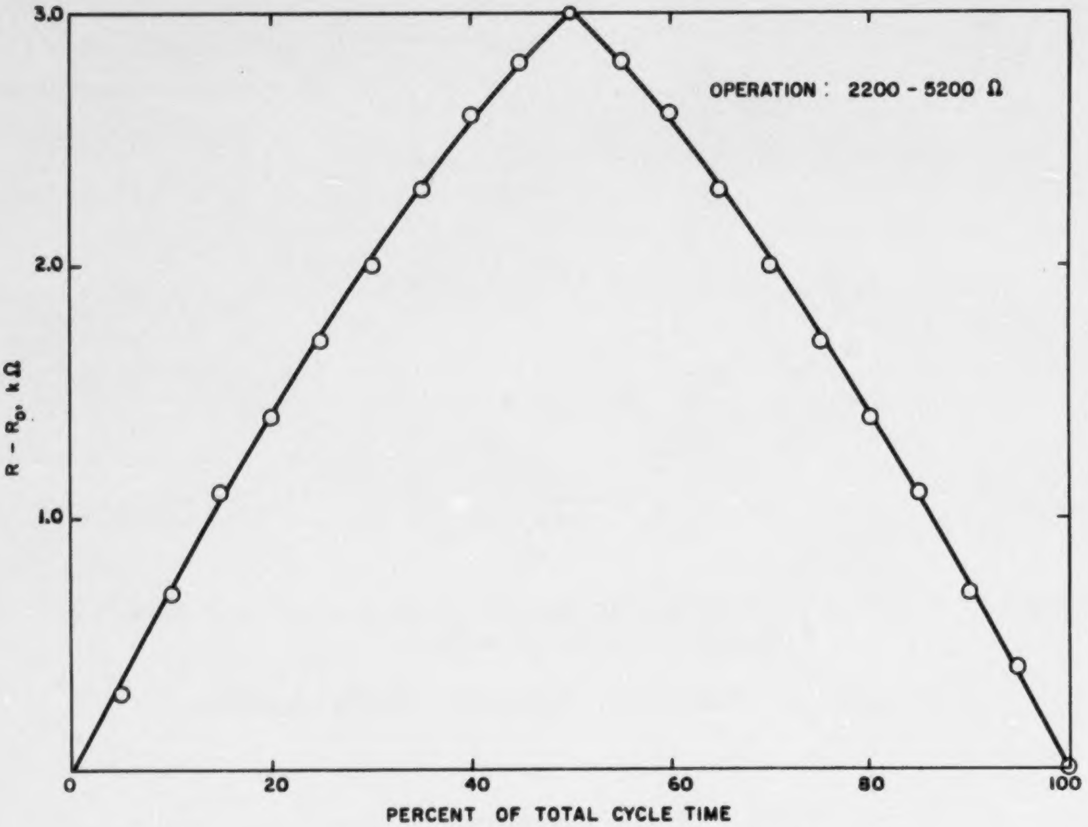


FIGURE 11. CIRCUIT RESISTANCE OF SAWTOOTH SPEED CONTROLLER

The nature and quantity of the nitrogen source is another feature which should be considered in the selection of an experimental substrate. The nitrogen-to-carbon ratio should be in the range of 0.33:1, and for simulation of domestic sewage approximately 50 percent of the nitrogen should be organic.

Glucose has been commonly used in biological waste treatment research [56,57], and this carbohydrate was screened as a possible substrate constituent during the early phases of this study. It was found, however, that the presence of glucose in a substrate stimulated the formation of dispersed growth cultures, uncharacteristic of conventional activated sludge. Furthermore it is very unlikely that glucose, or any other simple, readily metabolizable, low molecular weight organic compounds comprise a significant portion of the organic constituents contained in waste streams entering the majority of existing activated sludge plants. Biological activity in sewers and during primary treatment will certainly diminish the concentration of such constituents, because of the relative ease with which they are metabolized.

It was therefore felt that the principle carbon and energy source should be a high molecular weight, nonparticulate substance, with a high nitrogen content. It was essential that the substrate be nonparticulate so that membrane filtration could be used to remove cellular material from the effluent samples. In view of these limitations, egg albumen protein was selected as the major substrate constituent and beef extract was added as a source of growth factors. These two components were combined in a COD ratio of approximately 3:1. The substrate (synthetic sewage) was prepared in various strengths (Table IX) in which the ratio of the organic constituents was kept constant.

Bacterial growth in the feed reservoirs was prevented during long storage periods by autoclaving the feed reservoir, the delivery tubing, and all constituents, except the protein. In order to prevent denaturing this substance, a concentrated solution that had been dissolved in weak alkali (10 mℓ 4N NaOH in 500 mℓ) was slowly heated to 80° C and allowed to cool. Sixteen liters of hot tap water was added to the feed reservoir and after the contents of the reservoir had cooled, the media constituents were carefully added followed by 10 mℓ of 4N NaOH which raised the pH to about 10.5 and provided additional protection against bacterial contamination, but did not result in a highly buffered solution. A small quantity of phenolphthalein was included in the feed to provide visual indication of the pH of the reservoir contents since a drop in pH was an early indication that the feed material was seriously contaminated. Spot checks of reactor contents showed pH values in the range of 7.2 to 7.5.

Activated Sludge

Stock activated sludge cultures were obtained from the Valley Community Services District activated sludge plant at San Ramon, California and were maintained in a fill-and-draw laboratory unit on daily feedings of settled domestic sewage. Before using the sludge in an experiment it was passed through a No. 40 screen to remove debris and then thoroughly acclimated to the synthetic substrate by feeding it on a continuous basis for at least two cell residence periods (up to 40 days). Sludges developed on the synthetic sewage ranged from about 80 to 90 percent volatile and had a COD/VSS ratio of 1.35:1 and a N/VSS ratio of 0.091:1.

Laboratory Sampling and Analyses

Cell material was determined as volatile suspended solids using Whatman GF/C glass fiber filters and combustion for 30 minutes at 580° F [58]. The results of this technique for determination of activated sludge solids were found to be statistically identical to those obtained by using a Millipore HA (0.45 μ) membrane filter. The coefficient of variation of the method for activated sludge solids was 2.6 percent compared with 3.1 percent for the membrane filtration technique.

TABLE IX

COMPOSITIONS OF SYNTHETIC SEWAGES

Type	Constituent	Quantity	Concentration mg/ℓ	COD mg/ℓ
D-I	Egg albumen (Difco)	2.56 g	142	150
	Beef extract (Difco)	0.81 g	44.8	31
	K_2HPO_4		52.2	
	Ethanolic phenolphthalein soln.	1.0 mℓ		35
	Total COD			216
	NaOH (4N)	10.0 mℓ		
	Tap water	to 18.0 ℓ		
D-II	Egg albumen (Difco)	5.12 g	284	300
	Beef extract (Difco)	1.62 g	89.6	62
	K_2HPO_4		52.2	
	Ethanolic phenolphthalein soln.	2.0 mℓ		70
	Total COD			432
	NaOH (4N)	10.0 mℓ		
	Tap water	to 18.0 ℓ		
D-III	Egg albumen (Difco)	7.68 g	426	450
	Beef extract (Difco)	2.43 g	134.4	93
	K_2HPO_4		52.2	
	Ethanolic phenolphthalein soln.	3.0 mℓ		105
	Total COD			648
	NaOH (4N)	10.0 mℓ		
	Tap water	to 18.0 ℓ		

Suspended solids (SS) and volatile suspended solids (VSS) were determined daily on a 10-mℓ sample taken from the center of each reactor with a wide tip pipette 10 minutes after the reactor had been cleaned with a bristle brush. During sludge mass balance experiments, daily SS and VSS analyses were also conducted on the contents of the waste and effluent collection vessels. The volumes of liquid contained in these vessels were recorded so that the total mass of wasted cells could be computed.

Substrate. Soluble COD was used as the substrate parameter in these studies. The soluble effluent constituents were separated from cell material passing the liquid through a 0.45-μ membrane filter (Millipore HA) contained in a special holder mounted on the tip of a syringe. The syringe was fitted with a filling unit (Becton-Dickenson Co.) which was constructed with check valves to assure passage of the fluid in one direction only. Membrane filters were soaked in distilled water for 24 hours and rinsed immediately prior to use. At least 10 mℓ of sample was filtered before an aliquot was taken for analysis. Greater than 98 percent of the feed COD could be recovered after filtration by this technique and therefore feed samples were not filtered prior to analysis.

The COD procedure was a modification of the SERL method [59] developed during this research for the analysis of small volume samples (< 10 mℓ) having low COD values (< 50 mg/ℓ). The precision of this technique was excellent – in the COD range of 11.8 - 62.5 mg/ℓ the standard deviation was about 1.4 mg/ℓ, while in the COD range of 207 - 484 mg/ℓ the standard deviation was about 3.7 mg/ℓ.

During the steady state experiments daily effluent samples were taken in the morning at a time when the reactors had been running undisturbed for at least 12 hours. Feed samples for COD analysis were taken at least twice a week.

In the transient experiments a rotating fraction collector with a variable interval (10-30 min) setting (Misco) was used to collect the effluent stream. Since a maximum of 12 hours elapsed before the fractions were emptied from their collection tubes and filtered, a solution of $HgCl_2$ (2 g/ℓ) was added to each tube of the fraction collector so that the concentration upon filling the tube would be at least 10 mg Hg^{++}/ℓ.

Data Accumulation and Analysis

From the raw data on flow rate, volatile suspended solids, and COD, calculations were made to provide the daily operational characteristics q_z and D_c for each reactor. In the steady state experiments, the reactors were operated for a period of at least 2 D_c^{-1} or 40 days, whichever was shorter. Under the former conditions, at least 85 percent of cells in the reactor were native to the system and thus the probability of attaining balanced growth was improved.

The calculated results from several days of operation were averaged for each operating point and the averages were plotted according to Equation 30 to evaluate Y_Z and k_d and in the form of Equation 74 to determine k_o and c_1.

During the step transient experiments a continuous record was kept of the effluent soluble COD variations, while sludge concentration measurements were made daily. From a smoothed plot of the effluent soluble COD data the maximum value, $<s_1>$, was established, and the sludge concentration at this time was estimated from interpolation of the daily analytical results. From these data the asymptotic value of the specific substrate metabolism rate, $<q_Z>$, was calculated for each experimental run, and the results were plotted according to Equation 90 to evaluate the overall rate coefficient under transient input conditions.

A series of transient experiments were carried out to determine the variation of the endogenous metabolism coefficient as a function of growth rate. In these experiments the step function consisted of an immediate change in the cell dilution rate at constant feed conditions. Plots of Equation 97 provided values of the overall growth rate, G, and from values of D_c calculated from experimental data, the magnitude of k_d was established.

Periodic transient experiments were usually continued for three input cycles (three days). Samples were collected continuously using a fraction collector, and soluble COD analyses were performed on samples taken at stipulated intervals. Average SS and VSS for the experimental period were computed from the values of daily analyses. The COD data were then compared with theoretical values calculated by methods described in Chapter III (Equations 113a, 113b, 117a, and 117b).

PILOT-PLANT EXPERIMENTS

The activated sludge pilot plants used in this research are part of the experimental waste treatment facility at the Sanitary Engineering Research Laboratory. Two units were operated in parallel (Figure 12). Each unit was equipped with variable speed feed and return sludge pumps and sludge wasting equipment. The experimental conditions used in the currently reported studies are summarized below:

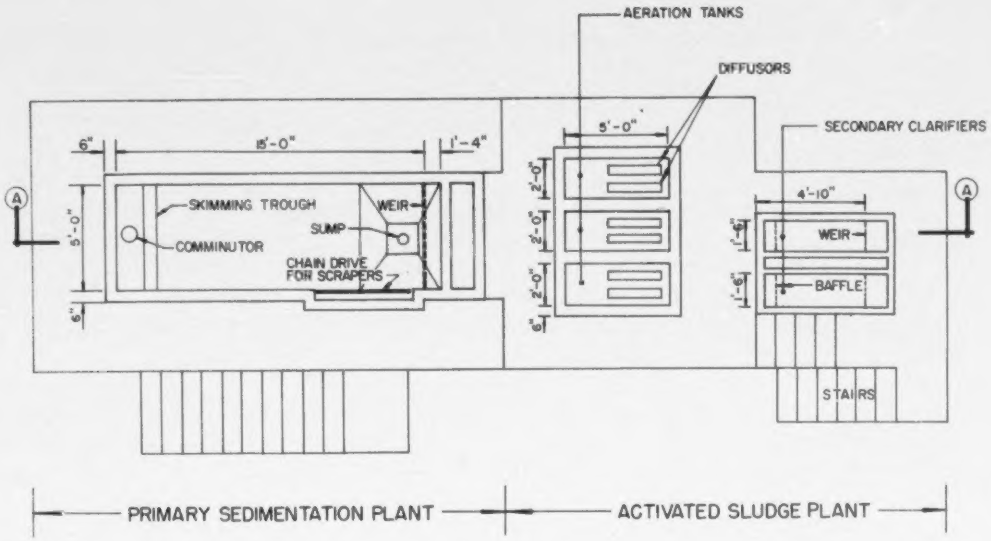
Aeration Tank Volume: 562 gal

Feed Rate: 480 to 4,750 gpd

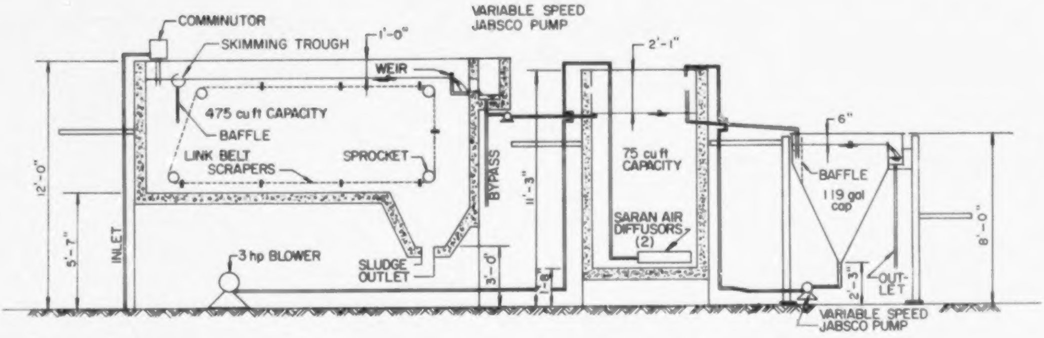
Dilution Rate: 0.85 to 8.46 per day

Mean Hydraulic Residence Time: 2.84 to 28.3 hr

MLVSS: 287 to 1,584 mg/ℓ .


AERATION TANKS
DIFFUSORS
5'-0"
SECONDARY CLARIFIERS
6"
15'-0"
1'-4"
A
SKIMMING TROUGH
WEIR
SUMP
COMMINUTOR
CHAIN DRIVE FOR SCRAPERS
5'-0"
6"
2'-0"
2'-0"
2'-0"
6"
4'-10"
1'-4"
WEIR
1'-6"
BAFFLE
A
STAIRS
PRIMARY SEDIMENTATION PLANT
ACTIVATED SLUDGE PLANT

PLAN


COMMINUTOR
SKIMMING TROUGH
1'-0"
VARIABLE SPEED JABSCO PUMP
2'-1"
WEIR
475 cu ft CAPACITY
BAFFLE
LINK BELT SCRAPERS
SPROCKET
BYPASS
75 cu ft CAPACITY
6"
BAFFLE
119 gal cap
12'-0"
5'-7"
INLET
3 hp BLOWER
SLUDGE OUTLET
3'-0"
11'-3"
1'-8"
SARAN AIR DIFFUSORS (2)
2'-3"
OUTLET
8'-0"
VARIABLE SPEED JABSCO PUMP

SECTION A-A

FIGURE 12. SEWAGE TREATMENT PILOT PLANT, RICHMOND FIELD STATION

The pilot plants were fed with domestic sewage obtained from a trunk sewer of the Richmond, California system and passed through a primary clarifier before being pumped to the activated sludge units. Sludge was wasted intermittently (each hour) by pumping directly from the aeration basins.

Influent and effluent samples were 24-hour composites collected by sampling devices which, every hour, discharged a sample of the flow to a refrigerated container. COD analyses were performed on the 24-hour composite feed and the membrane filtered effluent samples. A limited number of BOD analyses were made on influent and filtered effluent samples.

Suspended solids analyses (using the glass fiber filter method) were made daily on grab samples from the aeration basin and the waste sludge stream and on the composite effluent samples.

Flow rates and sludge wasting rates were established by daily measurements of the volumetric discharge from the feed and waste pumps.

Calculations from the daily monitoring data were made to establish the values of the specific substrate metabolism rate, q_z, and the cell dilution rate, D_c. The data were then grouped for periods of constant input-output conditions, and averages calculated. Statistical analyses were carried out using the average data, and the parameters Y_z, k_o, and c_1 were evaluated. Even though the pilot-plant data cover only a limited range of loadings, they provide support for the results obtained in the laboratory.

INITIAL UPTAKE EXPERIMENTS

Continuous Flow

Studies of initial uptake were first attempted by adding slugs of feed material to continuous flow laboratory reactors but due to the time lag and dispersion in the cell separators the results were not entirely satisfactory.

Batch Experiments

Activated sludge was maintained in continuous flow reactors and at the commencement of a batch experiment the reactor contents were apportioned into beakers. A measured quantity of substrate was then added to each beaker, and samples were withdrawn at specified time intervals. Each was immediately centrifuged at low speed for five minutes and the supernatant was decanted and refrigerated at 4° C. At the conclusion of the experiment, the samples were filtered through 0.45-μ membrane filters and the residual soluble COD determined as a function of time.

Data Analysis

The magnitude of initial uptake was estimated for both the continuous flow and batch experiments by extrapolation of concentration – time plots back to "zero time." Because of the plug flow behavior of the cell separators in the continuous flow experiments, the hydraulic residence time distribution of the units was determined by dye tracer studies so that the breakthrough time (or time lag) of the system could be established. The breakthrough time was selected as "zero time," because the conditions of the effluent at this point most closely represented the conditions in the reactor immediately after the addition of the substrate pulse.

In the batch experiments the conclusion of initial uptake could be recognized by a distinct break in the plot of soluble COD and time. The points following this break were fitted with a straight line, which was then projected back to "zero time." The difference between the projected and calculated soluble COD values amounted to the substrate assimilated during initial uptake. The residual COD (the equilibrium value) was corrected for end product COD on the basis of the COD of the feed, z_o, used to sustain the sludge prior to the experiments (Equation 66).

Plots were prepared from these data that showed the variation of specific initial removal (mass of substrate COD removed per unit mass of TSS) with residual substrate COD concentration, and the "adsorption" coefficient, ϵ, was obtained from the slope of this line.

V. RESULTS

LABORATORY STEADY STATE EXPERIMENTS

The results of steady state laboratory experiments designed to evaluate the yield coefficient, Y_z, and the endogenous rate coefficient, k_d, are presented in Table X. These data are averages of the daily values which are summarized in Appendix C. A plot of cell dilution rate, D_c, and specified substrate removal rate, q_z, is shown in Figure 13. From the statistically fitted line, Y_z is determined as 0.46 lb VSS/lb COD and k_d as 0.08 day^{-1}.

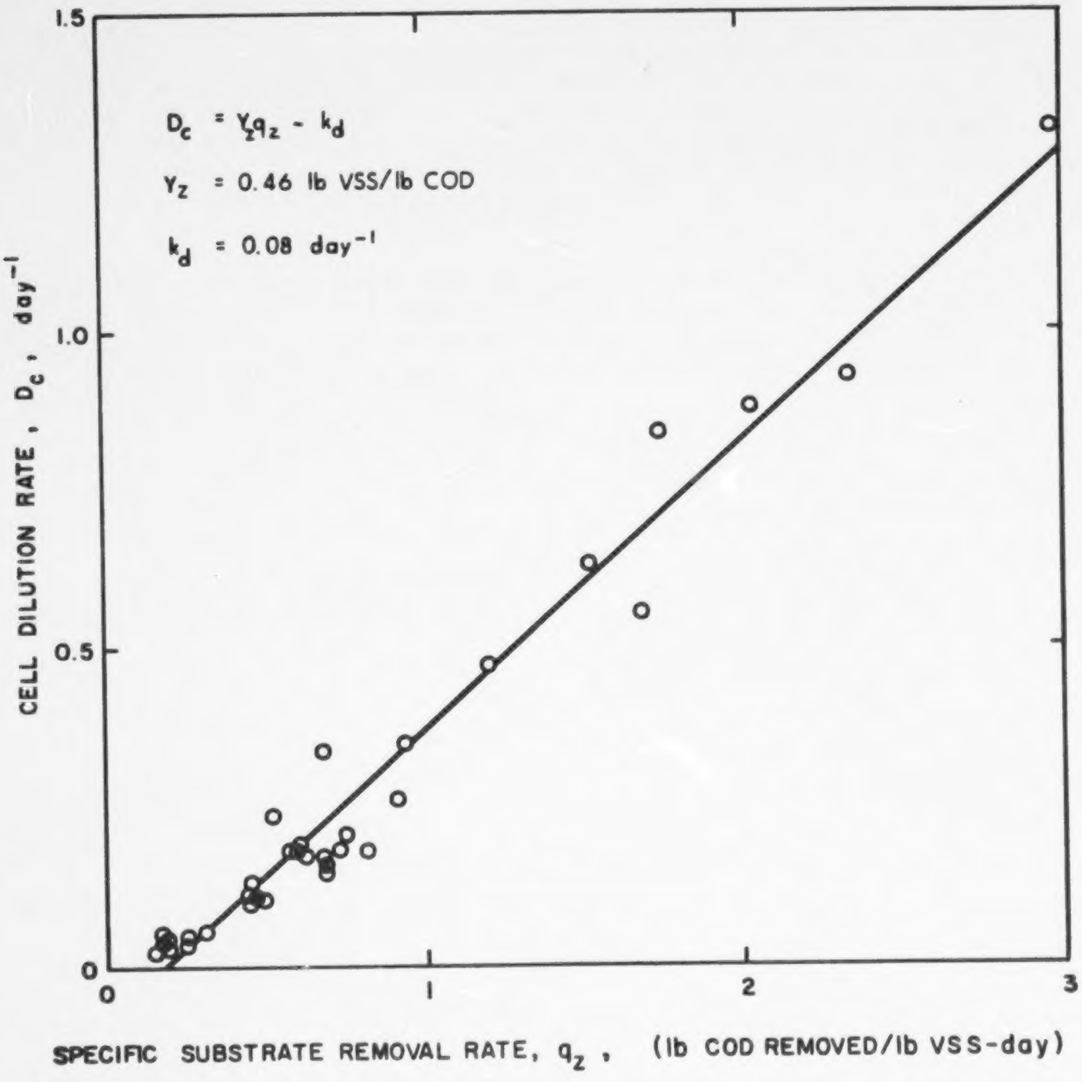


FIGURE 13. CELL CONTINUITY EQUATION FOR LABORATORY DATA

TABLE X

RESULTS OF LABORATORY STEADY STATE EXPERIMENTS FOR EVALUATION OF Y_z AND k_d

Run No.	z_0 mg/ℓ	z_1 mg/ℓ	D day^{-1}	x_1 mg/ℓ	D_c day^{-1}	q $\frac{\text{lb COD removed}}{\text{lb VSS-day}}$
65-21	373	27.1	2.98	5077	0.032	0.20
65-22	475	36.0	2.74	4922	0.036	0.25
65-23	383	52.3	2.83	3016	0.053	0.31
65-24	401	36.8	3.10	2530	0.099	0.45
65-25	220	17.1	3.21	3726	0.045	0.18
65-26	580	71.2	3.13	3513	0.112	0.45
65-27	212	23.1	3.00	3436	0.029	0.16
65-27	254	38.0	3.15	1395	0.129	0.45
65-28	417	35.4	3.00	5963	0.040	0.19
65-29	239	30.7	3.02	1335	0.103	0.49
65-29	412	42.9	3.03	2437	0.102	0.46
65-30	575	42.0	3.06	2615	0.169	0.62
65-31	415	32.9	3.07	1939	0.189	0.61
65-32	254	25.1	3.06	1368	0.234	0.52
65-33	255	28.4	3.00	898	0.208	0.75
65-34	820	236	3.07	1120	0.555	1.68
65-35	614	127	3.01	1577	0.349	0.94
65-35	634	228	3.00	761	0.840	1.74
65-36	426	131	3.04	590	0.629	1.52
65-37	220	69	2.91	598	0.337	0.68
65-38	840	98	3.02	2477	0.259	0.91
65-39	611	64	3.06	2313	0.181	0.73
65-39	205	27.8	3.05	975	0.170	0.68
65-40	415	43.3	3.13	1705	0.158	0.69
65-41	209	35.3	3.01	796	0.150	0.69
65-41	418	54.2	3.00	1340	0.180	0.82
S-1	213	14.0	3.01	3040	0.04	0.19
S-2	217	16.3	3.14	1090	0.18	0.58
S-3	418	28.2	2.96	980	0.47	1.20
S-4	622	53.2	3.21	805	0.93	2.34
S-5	184	11.9	2.91	1820	0.04	0.25
S-6	414	23.8	3.34	2040	0.18	0.58
S-8	412	40.0	5.00	745	0.88	2.03
S-8	412	40.7	5.00	570	1.32	2.97

Although the data points can be fitted with a straight line, at low q_z values the points tend to curve toward the origin rather than intercept the abscissa. Thus in the range of operation of conventional activated sludge ($q_z < 1.0$), the absolute values of Y_z and k_d would be smaller if the relationship $D_c = Y_z q_z - k_d$ is valid. One explanation for the deviation of this curve from linearity is that k_d is not a constant and indeed experimental data reported later tend to support this contention.

Kinetic data for evaluation of the overall rate coefficient, k_o, and the limiting soluble COD residual, c_1, are presented in Table XI. This table contains only a portion of the experimental data included in Table X because during the early phases of this research, the synthetic sewage was prepared so that one of the organic constituents was not present in constant proportion when substrates of various strengths were prepared. This difference in substrate character did not influence the values of either Y_z or k_d but it was found that the overall rate coefficient was quite substrate dependent.

In addition to the data in Table XI, it was possible to use some experimental results from batch initial uptake studies for the evaluation of k_o and c_1. Thus in Figure 14, which is a plot of q_z/z_o and z_1/z_o according to Equation 74, the points on the abscissa were derived from batch initial uptake studies (Table XII).

A two-variable least-squares analysis of the data in Figure 14 gave the following values for k_o and c_1:

$$k_o = 0.172\ \ell/\text{mg-day}$$

$$c_1 = 0.0557.$$

The correlation coefficient from the least-squares analysis was greater than 0.96, and a t-test analysis indicated with much greater than 99 percent confidence that the correlation is different from zero.

Nevertheless it is possible that a more accurate representation of the data might be a curved line with a slope increasing with the value of q_z/z_o as was anticipated by the example shown in Figure 3. It is not possible to determine whether this is indeed so because of the sparsity of data points in the higher range of q_z/z_o values; however data from the SERL pilot plant and from the field shows that this is indeed the case.

TABLE XI

EVALUATION OF k_o AND c_1 FROM LABORATORY STEADY STATE RESULTS

Run	COD, mg/ℓ		MLSS mg/ℓ	MLVSS x_1 mg/ℓ	Specific Substrate Removal Rate, q lb COD/lb VSS-day	Dilution Rate, D day^{-1}	$\left(\frac{q_z}{z_o}\right)$ x 10^3	$\left(\frac{z_1}{z_o}\right)$
	Influent z_o	Soluble Effluent z_1						
66-01	236	18.1	1034	952	0.67	2.95	2.84	0.077
66-03	211	15.2	1457	1338	0.43	2.94	2.04	0.072
66-04	216	16.7	1405	1306	0.47	3.13	2.18	0.077
66-05	213	14.0	2296	1926	0.31	3.02	1.46	0.066
66-06	419	24.2	1923	1753	0.66	2.95	1.58	0.058
66-07	420	25.1	3032	2635	0.46	3.08	1.10	0.060
66-08	605	45.2	2434	2234	0.76	3.17	1.26	0.075
66-25	218 218	15.1 18.3		692 374	0.96 1.63	3.18 3.03	4.40 7.47	0.069 0.084
66-32	190	21.9		322	3.32	6.20	17.47	0.126
S-1	213	14.0		3040	0.19	3.01	0.91	0.066
S-2	217	16.3		1090	0.58	3.14	2.69	0.075
S-3	418	28.2		980	1.20	2.96	2.87	0.068
S-4	622	53.2		805	2.34	3.21	3.76	0.086
S-5	184	11.9		1820	0.25	2.91	1.34	0.065
S-6	414	23.8		2040	0.58	3.34	1.39	0.058
S-8	412 412	40.3 41.2		745 570	2.28 3.14	5.00 5.00	5.54 7.63	0.098 0.100

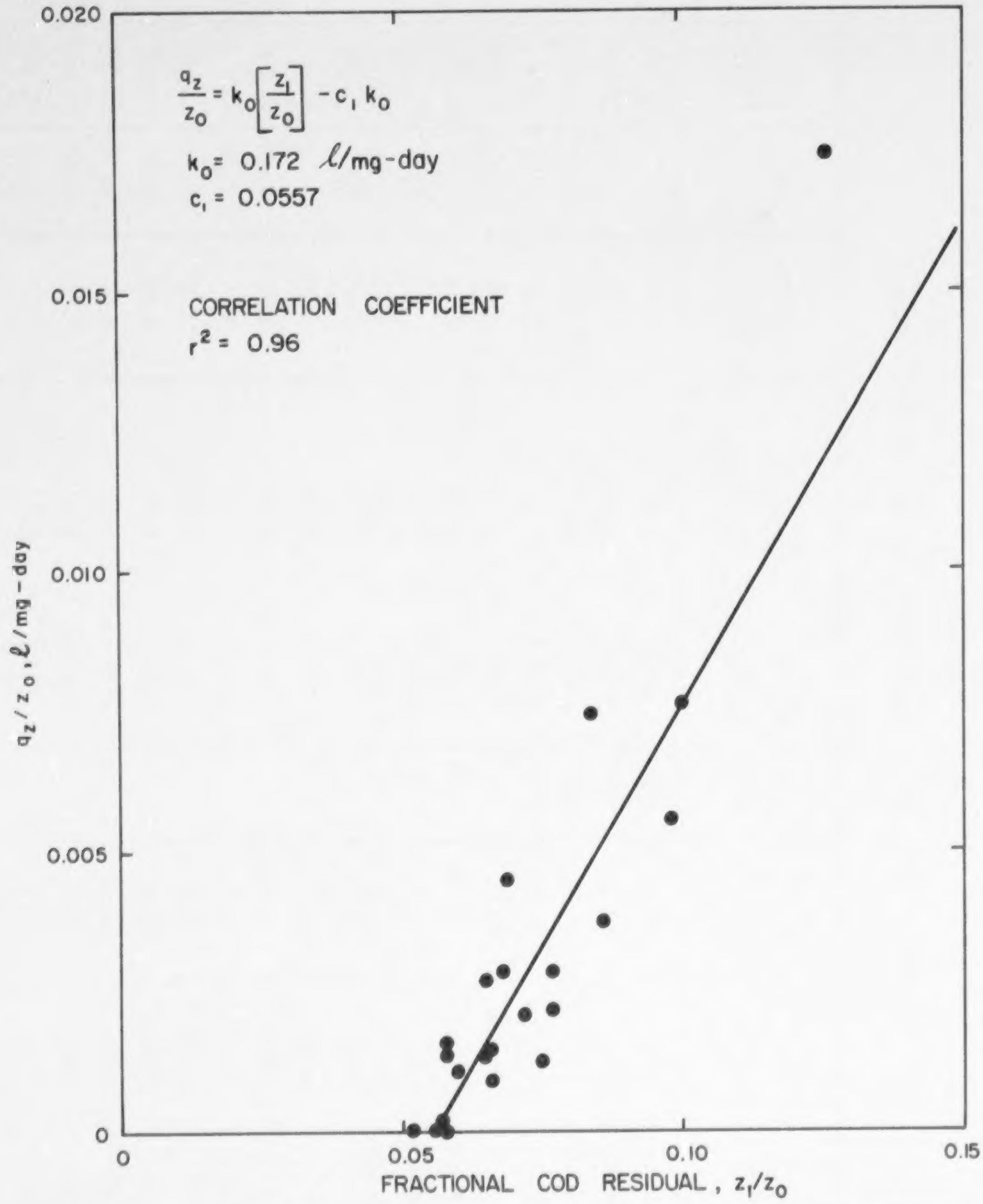


FIGURE 14. KINETIC PLOT FOR STEADY STATE LABORATORY DATA

TABLE XII

LIMITING RESIDUAL COD VALUES[a] FROM INITIAL UPTAKE EXPERIMENTS

Run Identification	Pulse COD mg/ℓ	Initial Feed COD mg/ℓ	Residual COD mg/ℓ	Fractional Residual z_1/z_0
66-20	263	103	20.0	0.056
66-19	198	102	17.0	0.057
	343	102	23.3	0.052
	650	102	43.4	0.058

[a] Analyses made 24 hr after feeding.

PILOT-PLANT AND FIELD-SCALE EXPERIMENTS

A summary of the kinetic data obtained from the SERL pilot plant during this study is presented in Table XIII together with kinetic data obtained by Jenkins and Menar [23] from the same pilot plant. The operating points obtained during the present study represent a minimum of 15 days of operation.

Figure 15 is a plot of the parameters D_c and q_z for the organic loading range covered by this study. This plot differs from Figure 13 in three respects: 1) the concave upward trend is not displayed; 2) the intercept strikes the ordinate above the origin, indicating a k_d of -0.05 day^{-1}; and, 3) the slope of the least squares line, Y_z, is substantially less over the same range of growth rates. For the SERL pilot plants, Y_z was 0.33 lb VSS/lb COD removed. The experiments of Jenkins and Menar [23] indicate an identical value for Y_z in this range of loadings and give a k_d value of 0.001 day^{-1}.

The absence of curvature in the slope and the negative value of k_d in the data from the current study may possibly be related to the limited amount of data collected and to inaccuracies inherent in sampling from a large pilot plant. For example, in these experiments no account was taken of the mass of sludge contained in the final clarifier, the assumption being that this quantity of sludge was small compared to the mass of sludge in the aeration basin. This assumption would cause the greatest error at low growth rates, because proportionately greater quantities of sludge tended to accumulate in the clarifiers under these conditions. Thus the calculated value of q_z would be low, and a high estimate of D_c would be made which would tend to shift the plot upwards and to the left.

TABLE XIII

SERL PILOT PLANT KINETIC DATA SUMMARY

Operating Dates	Feed COD z_o mg/ℓ	Effluent Soluble COD z_1 mg/ℓ	q_z $\frac{\text{lb COD Removed}}{\text{lb MLVSS-day}}$	D_c day^{-1}	$\frac{10^3 q_z}{z_o}$ ℓ/mg-day	$\frac{z_1}{z_o}$	MLVSS x_1 mg/ℓ	Hydraulic Residence Time, θ hr
5/26/66 - 6/09/66[a]	254	36	0.39	0.135	1.54	0.141	1290	11.0
6/10/66 - 6/24/66[a]	250	44.1	1.04	0.44	4.2	0.176	865	5.6
5/26/66 - 6/22/66[a]	252	45.3	3.57	1.25	14.2	0.180	513	2.9
5/26/66 - 6/09/66[b]	255	36.2	0.32	0.11	1.26	0.142	1310	10.0
6/16/66 - 7/11/66[b]	248	47.7	0.88	0.28	3.55	0.192	686	6.4
6/07/66 - 7/11/66[b]	252	41.1	2.97	0.98	11.8	0.163	496	2.8
8/01/66 - 8/09/66[b]	319	61.6	5.60	2.28	17.6	0.193	479	1.9
8/02/66 - 8/11/66[b]	295	51.6	5.78	2.58	19.6	0.175	463	1.9
8/17/66 - 9/07/66[b]	274	62.7	13.7	6.94	50.0	0.229	254	1.25
8/17/66 - 9/07/66[b]	280	61.6	15.5	9.35	55.3	0.220	226	1.2
8/19/66 - 8/30/66[b]	252	63.0	20.8	11.4	82.5	0.250	218	1.0
9/09/66 - 9/30/66[b]	245	74.7	42.5	28.2	174.	0.305	91	0.9
8/14/67 - 11/1/67[c]	274	61.2	15.1		55.0	0.223		

[a] This study.

[b] Jenkins and Menar [23].

[c] Unpublished data, Weddle [60].

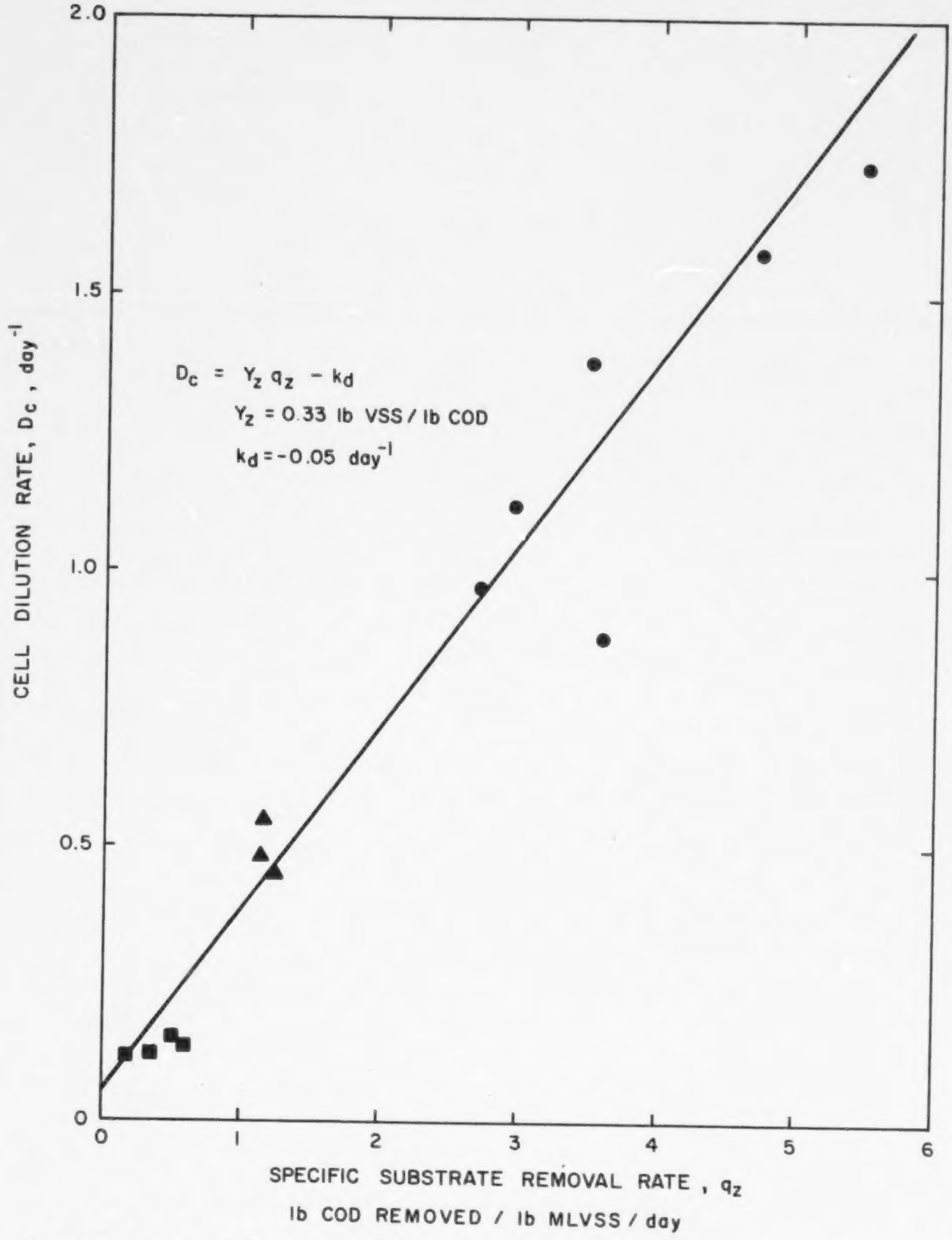


FIGURE 15 . CELL CONTINUITY EQUATION FOR PILOT PLANT DATA

The lower value of Y_z for activated sludge plants treating domestic wastes is substantiated by field data obtained from the sources listed in Table XIV. Data from these activated sludge plants are summarized in Table XV and plotted in Figure 16, giving a yield coefficient of 0.336 lb VSS/lb COD removed and an endogenous metabolism coefficient of 0.016 day^{-1}.

TABLE XIV

PLANTS FROM WHICH FIELD DATA
HAVE BEEN COLLECTED

Location	Type	Reported By
Pomona, Calif.	Standard Rate High Rate Activated Sludge Water Reclamation	Jenkins and Garrison [24]
Whittier Narrows, Los Angeles, Calif.	Standard Rate Activated Sludge Water Reclamation	Haas and Pearson [61] Garrison [62]
San Ramon, Calif.	Standard Rate Activated Sludge	Jenkins [63]
Jackson, Miss.	Extended Aeration Activated Sludge (Air National Guard Armory)	Middlebrooks [64]
National Sanitation Foundation	Extended Aeration Activated Sludge	Goodman [65]

The substrate-linked kinetic data from the SERL pilot plants are plotted in Figure 17. The points display a greater degree of scatter than the laboratory data because of the day-to-day variability of the waste stream fed to the pilot plant. Nonetheless, it is possible to demonstrate significant differences between the pilot plant and laboratory results.

The limiting soluble COD value is approximately 12 percent of the influent COD, a value that is more than twice that found for the synthetic substrate used in the laboratory experiments. It is possible that the higher limiting soluble COD value is caused by the presence of nondegradable constituents in the feed, but on the other hand it is not inconceivable that a greater quantity of metabolic end products accumulate in activated sludge plants treating domestic sewage. These data serve to emphasize the caution with which COD measurements should be interpreted when considering process efficiency.

TABLE XV

FIELD KINETIC DATA SUMMARY

Location	z_o mg/ℓ	z_1 mg/ℓ	q_z $\frac{\text{lb COD Removed}}{\text{lb MLVSS-day}}$	$\frac{10^3 q_z}{z_o}$ ℓ/mg-day	z_1/z_o
Whittier Narrows, L.A., Calif. [61]	242	39	0.575	2.4	0.161
Whittier Narrows, L.A., Calif. [62]	277	33	0.55	2.0	0.119
National Sanitation Foundation [65]	224	33	0.074	0.33	0.146
San Ramon, Calif. [63]	294	45	0.238	0.81	0.153
Jackson, Miss. [64]	638	31	0.091	0.14	0.049
Pomona, Calif. [24]	247	34	1.21	4.9	0.139
Pomona, Calif. [24]	296	27	0.65	2.2	0.091

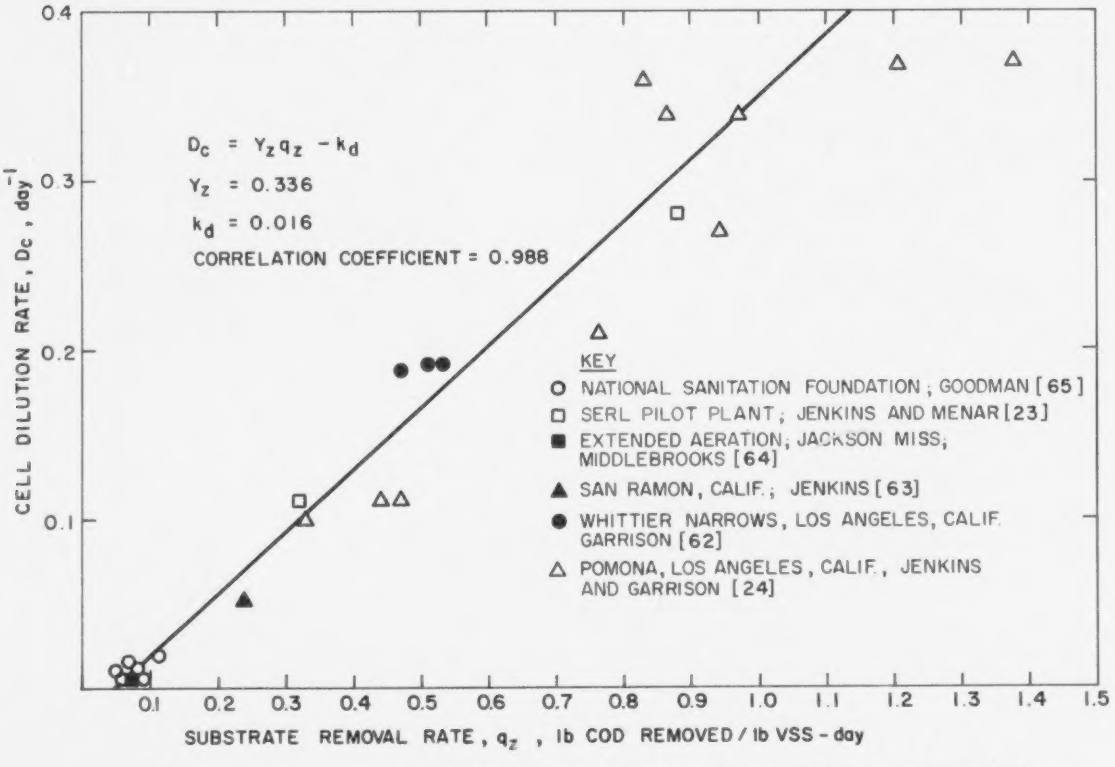


FIGURE 16. CELL CONTINUITY EQUATION FOR FIELD DATA

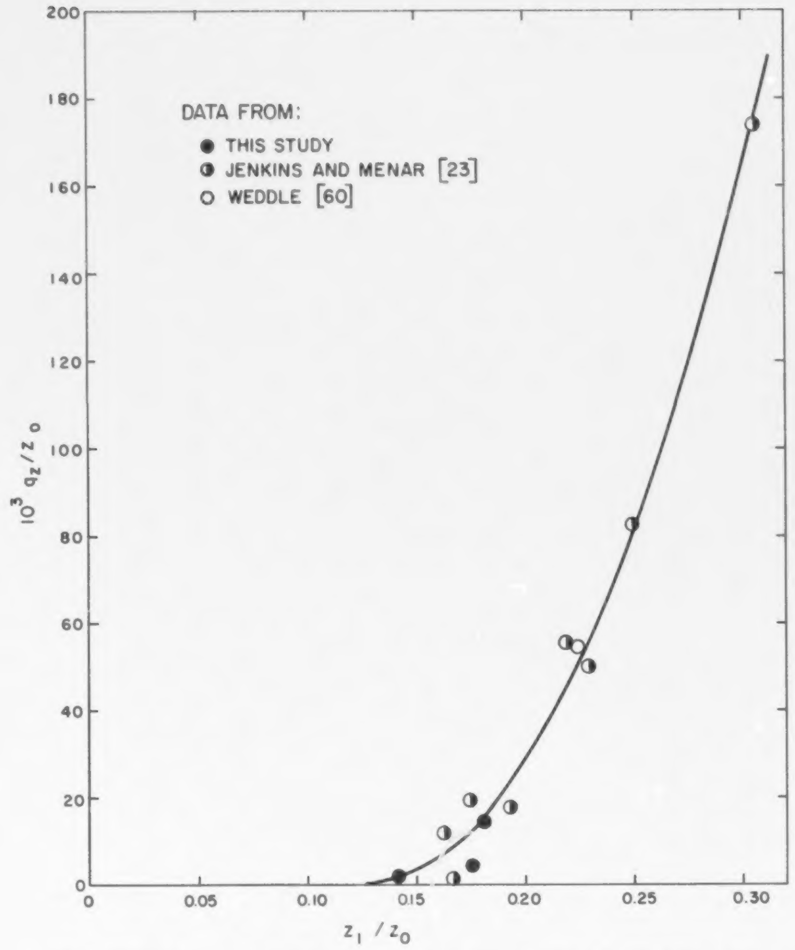


FIGURE 17. KINETIC PLOT OF SERL PILOT PLANT DATA

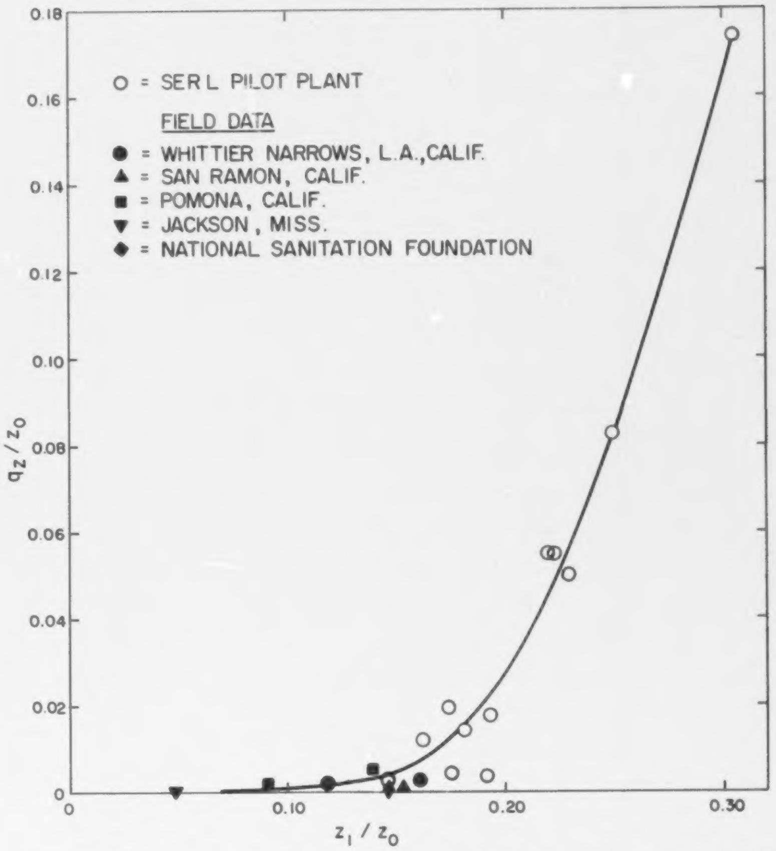


FIGURE 18. KINETIC PLOT FOR SERL PILOT PLANT AND FIELD DATA

The most significant aspects of Figure 17 are the slope of the fitted line, which is equivalent to the overall rate coefficient, k_o, and the curvature of this line. At higher values of the parameter q_z/z_o (which correspond to higher organic loading rates), the slope of the line (the value of k_o) is approximately 1.5 ℓ/mg-day – a value substantially greater than that obtained in the laboratory experiments. Moreover, the curvature of the line is concave upwards, which is opposite to the behavior predicted by the Monod relationship, but is consistent with predictions made in Chapter III of this report (see Figure 3).

Field data from the sources listed in Table XIV are plotted in Figure 18, together with the SERL pilot-plant data. Although none of the field-scale activated sludge plants were operated at loading rates (q_z) greater than 2.2 lb COD removed/lb MLVSS-day, the data are in good agreement with those obtained from the SERL pilot plants. Figure 18 clearly demonstrates the feasibility of using much greater organic loading rates in the activated sludge process without much loss of attainable COD removal efficiency. This conclusion is reflected by the calculations presented in Table XVI in which the data in Figure 18 have been used to predict the effluent soluble COD from activated sludge plants treating domestic sewage at high loading rates.

TABLE XVI

PREDICTED PERFORMANCE OF ACTIVATED SLUDGE PLANTS TREATING DOMESTIC SEWAGE

Influent COD, z_o mg/ℓ	Loading Rate, q_z $\frac{\text{lb COD Removed}}{\text{lb VSS-day}}$	q_z/z_o ℓ/mg-day	z_1/z_o	Effluent Soluble COD, z_1, mg/ℓ
250	2.5	0.01	0.168	42.0
	5.0	0.02	0.188	47.0
	10	0.04	0.214	53.5
	15	0.06	0.233	58.2
	20	0.08	0.248	62.0
	25	0.10	0.262	65.5

From this table it can be seen that at a q_z of 10, a loading at least ten times greater than employed in practice, the COD removal efficiency is 78.5 percent, only 8 percent lower than that predicted for a plant loaded at a more conventional q_z of 1.0 lb COD/lb VSS-day. It is recognized, however, that higher loading (growth) rates invariably result in dispersed sludge cultures, which are not readily separated in gravity clarifiers. Therefore, at higher loading rates the development of improved cell separation devices will be imperative if the potential of the activated sludge process is to be realized.

INITIAL UPTAKE EXPERIMENTS

The results of continuous flow and batch initial uptake experiments are presented in Table XVII and in Figures 19 and 20. The presence of small amounts of insoluble material in the synthetic substrate had no effect on the initial uptake as evidenced by Figures 19 and 20 which also show an initial uptake experiment for a synthetic feed which had been filtered through a 0.45-μ membrane filter. Membrane filtration itself suggested that the initial uptake results for filtered and unfiltered substrates would be similar since filtration removed only 2.3 percent of the substrate COD.

The initial uptake data presented in Figure 19 show substantial agreement in their limiting slopes but the maximum "sorptive" capacity varies over a wide range from approximately 0.04 to 0.08 mg COD/mg MLVSS.

The most common pulse transient situations encountered in CSTR activated sludge systems involve substrate COD variations of less than 100 mg/ℓ. By using the more reliable batch initial uptake data (Figure 20), the specific initial removal or maximum "sorptive" capacity in this region is determined from the limiting slope as $\epsilon = 5 \times 10^{-4}$ ℓ/mg. In these batch experiments the maximum sorptive capacity amounted to approximately five percent of the total substrate COD, which is substantially less than reported by many previous investigators, but is consistent with the physical properties of the substrate used in this research and with the theoretical rationale developed in Chapter III.

TRANSIENT STATE DETERMINATION OF ENDOGENOUS RATE COEFFICIENT

Continuous flow transient conditions make it possible to assess the magnitude of k_d without the parametric influence of Y_z since the transient state determination of k_d depends solely on evaluating the cell dilution rate, D_c, a parameter which can be measured directly.

To determine k_d on a transient continuous flow basis three transient runs were analyzed using Equation 97, which is closely approximated by

$$\ln\left[\frac{x_1 - (x_1)_\infty}{(x_1)_0 - (x_1)_\infty}\right] = -\left(D_c + k_d\right)t \qquad (119)$$

because the term D/k_o is usually negligible.

The transient conditions used to derive values of k_d at different values of D_c were obtained from transitional periods between steady state experiments, and the terminal conditions were established by visual fit of the data. Even though such a visual fit may introduce some subjectivity into the conclusion, it can be clearly demonstrated that the endogenous rate coefficient is not a constant (as has been assumed in this and other works), but rather it depends on specific growth rate of the microorganisms. Tables XVIII, XIX, and XX and Figures 21, 22, and 23 present data for the transient state determination of k_d. The indicated slopes in Figures 21 through 23 were fitted by eye because a

TABLE XVII

SUMMARY OF INITIAL UPTAKE EXPERIMENTS

Date - Run	COD of Pulse mg/ℓ	Residual Substrate COD[a] mg/ℓ	Initial Removal z_i mg/ℓ	MLVSS x_1 mg/ℓ	Specific Initial Removal z_i/x_1	Experimental Classification
3-24-66-16	196	80	116	3850	0.031	Continuous flow
3-29-66-16	395	200	195	3830	0.051	
3-31-66-16	582	338	244	3825	0.064	
4-05-66-16	95	30	65	4000	0.016	
3-23-66-17	202	113	89	1500	0.059	Continuous flow
3-28-66-17	401	260	141	1660	0.085	
3-30-66-17	588	433	155	1725	0.090	
4-04-66-17	101	47	54	1810	0.030	
4-06-66-17	158	93	65	1890	0.034	
4-07-66-18	190	92	98	1670	0.059	Continuous flow
4-11-66-18	152	70	82	1890	0.043	
4-13-66-18	95	36	59	1920	0.031	
4-14-66-18	386	218	168	2050	0.082	
4-15-66-18	97	42	55	2140	0.026	
4-25-66-19	230	145	85	1920	0.044	Continuous flow
4-30-66-19	198	127	71	1990	0.036	Batch reactor
5-03-66-19	346	265	81	1990	0.041	
5-05-66-19	652	560	92	2100	0.044	
5-04-66-20	269	226	43	1070	0.040	Batch reactor
6-15-66-29	51	33	18	1000	0.018	Batch reactor
	102	70	32	1000	0.032	
	155	127	28	1000	0.028	
	232	197	35	1000	0.035	
	309	259	50	1000	0.050	
6-15-66-29[b]	50	34	16	1000	0.016	Batch reactor
	101	64	37	1000	0.037	
	152	109	43	1000	0.043	
	226	178	48	1000	0.048	
	302	247	55	1000	0.055	

[a] Projected value at t = zero.

[b] Nutrient passed through membrane filter before use.

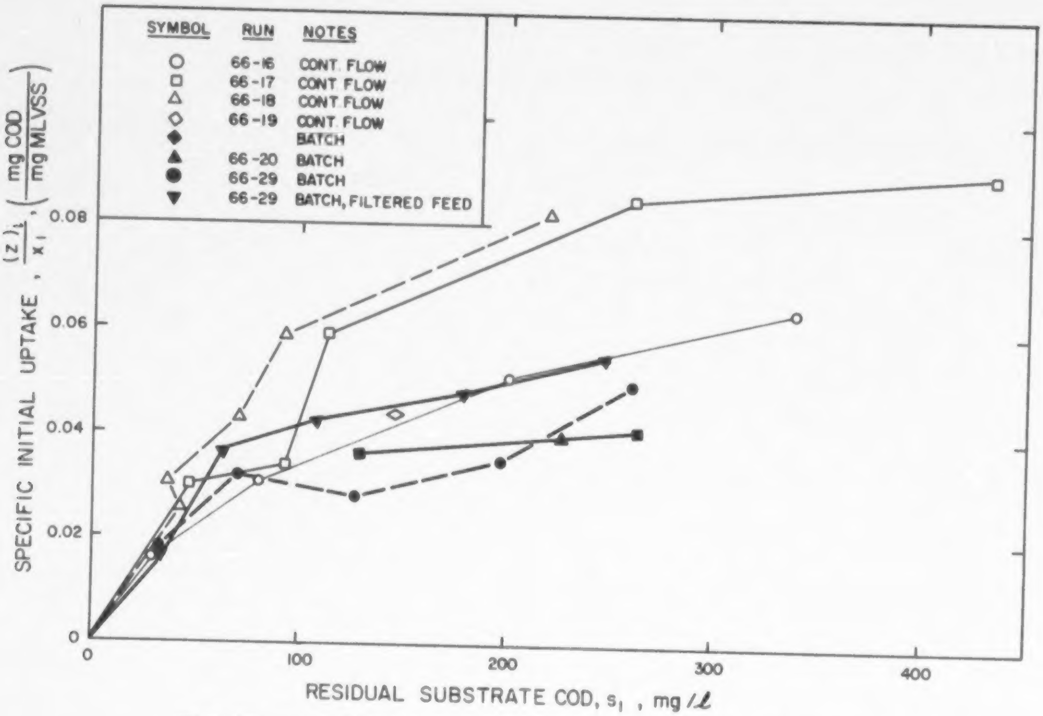


FIGURE 19. RESULTS OF INITIAL UPTAKE EXPERIMENTS

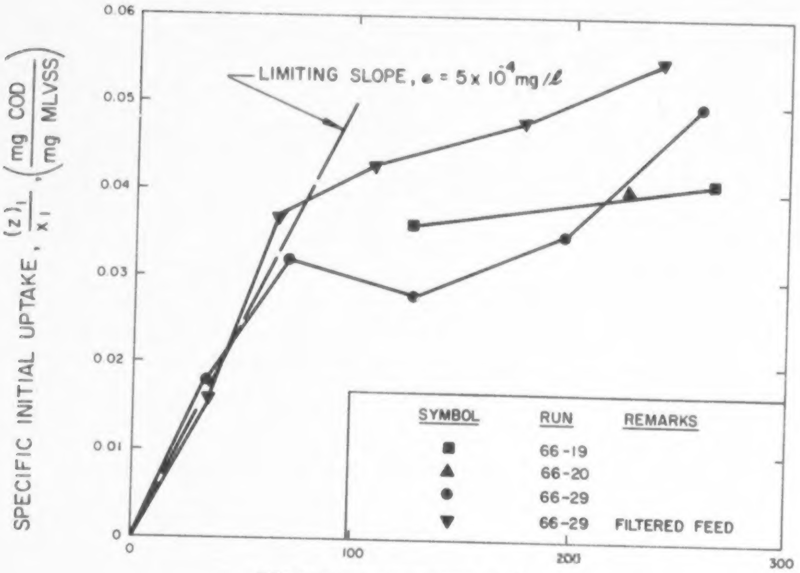


FIGURE 20. RESULTS OF BATCH INITIAL UPTAKE EXPERIMENTS

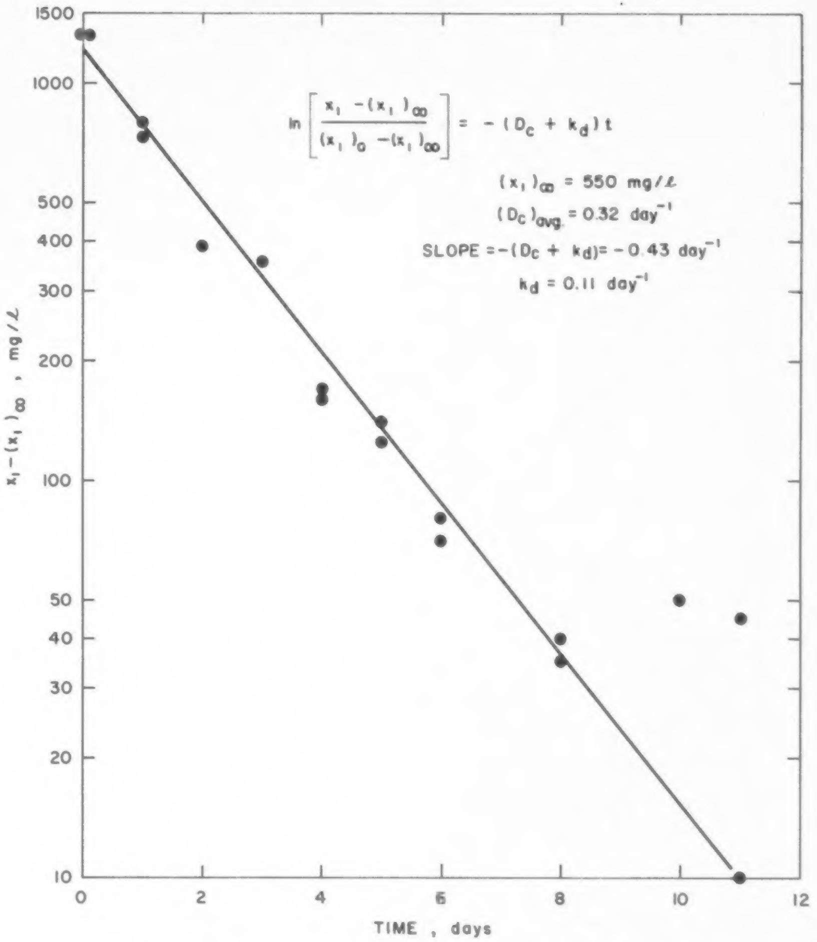


FIGURE 21. TRANSIENT STATE DETERMINATION OF ENDOGENOUS COEFFICIENT, RUN 65-16

TABLE XVIII

TRANSIENT STATE DETERMINATION OF ENDOGENOUS COEFFICIENT FOR RUN 65-16

$(x_1)_o = 1700$ mg/ℓ $(x_1)_\infty = 550$ mg/ℓ

Day	MLVSS, x_1 mg/ℓ	$x_1 - (x_1)_\infty$ mg/ℓ	Cell Dilution Rate, D_c, day^{-1}
0	1895	1345	-
	1895	1345	
1	1345	795	0.363
	1285	735	
2	940	390	0.344
3	905	355	0.301
4	720	170	0.303
	710	160	
5	690	140	0.300
	675	125	
6	630	80	0.308
	620	70	
7			0.307
8	585	35	0.317
	590	40	
9			0.386
10	600	50	0.267
11	560	10	
	595	45	0.367
Average			0.319

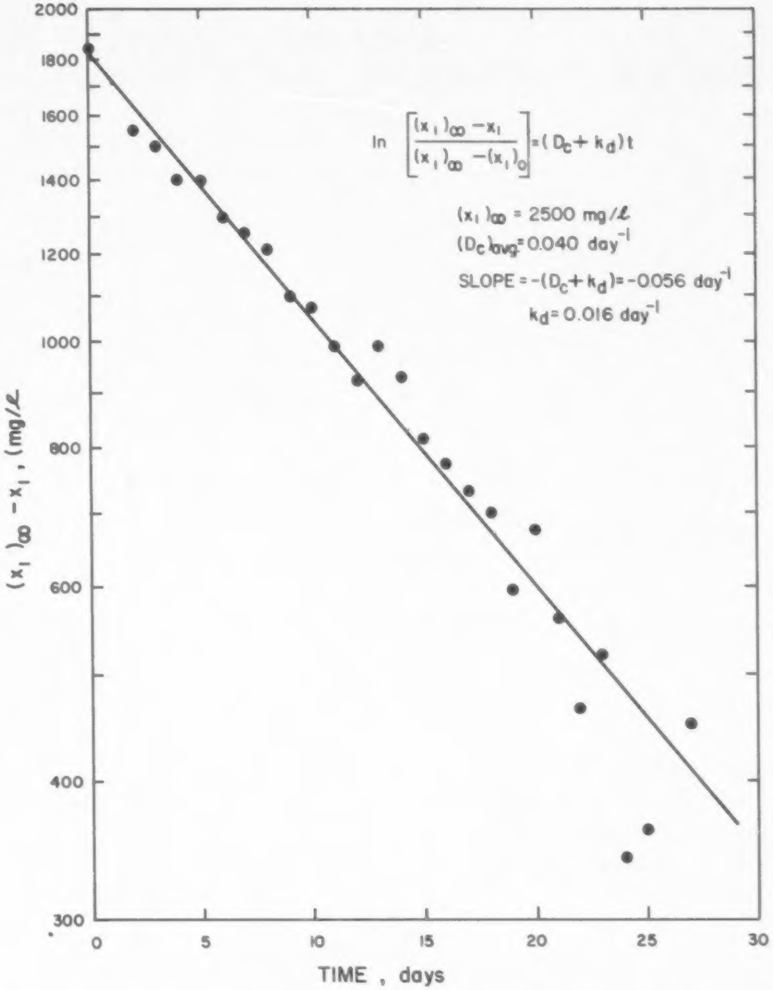


FIGURE 22. TRANSIENT STATE DETERMINATION OF ENDOGENOUS COEFFICIENT, RUN 65-20

TABLE XIX

TRANSIENT STATE DETERMINATION OF ENDOGENOUS COEFFICIENT FOR RUN 65-20

$(x_1)_0 = 655$ mg/ℓ $(x_1)_\infty = 2500$ mg/ℓ

Day	MLVSS, x_1 mg/ℓ	$(x_1)_\infty - x_1$ mg/ℓ	Cell Dilution Rate, D_c, day^{-1}
0	655	1845	0.048
1	795	1705	0.051
2	945	1555	0.043
3	995	1505	0.031
4	1100	1400	0.032
5	1100	1400	0.033
6	1205	1295	0.031
7	1245	1295	0.031
8	1290	1210	0.034
9	1405	1095	0.042
10	1425	1075	0.068
11	1505	995	0.063
12	1580	920	0.054
13	1510	990	0.071
14	1570	930	0.051
15	1685	815	0.032
16	1725	775	0.029
17	1770	730	0.037
18	1800	700	0.039
19	1905	595	0.029
20	1825	675	0.028
21	1940	560	0.028
22	2035	465	0.029
23	1980	520	0.031
24	2160	340	0.031
25	2140	360	0.034
26	-	-	0.036
27	2050	450	0.038
Average			0.040

TABLE XX

TRANSIENT STATE DETERMINATION OF ENDOGENOUS COEFFICIENT FOR RUN 65-27

$(x_1)_o$ = 3400 mg/ℓ $(x_1)_\infty$ = 1450 mg/ℓ

Day	MLVSS, x_1 mg/ℓ	$x_1 - (x_1)_\infty$ mg/ℓ	Cell Dilution Rate, D_c, day^{-1}
0	3400	1950	-
1	3220	1770	0.113
2	2940	1490	0.123
3	2660	1210	0.115
4	2560	1110	0.112
5	2360	910	0.124
6	2220	770	0.135
7	2160	710	0.124
8	1940	490	0.123
9	1940	490	0.140
10	1980	530	0.143
11	1760	310	0.136
12	1760	310	0.135
Average			0.128

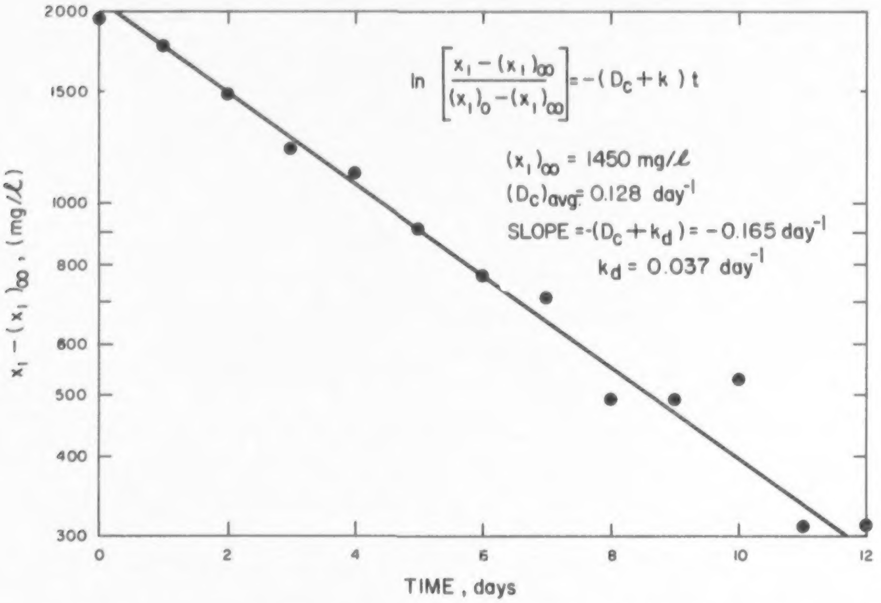


FIGURE 23. TRANSIENT-STATE DETERMINATION OF ENDOGENOUS COEFFICIENT, RUN 65-27

normal regression analysis would have weighted the ordinate values according to their logarithms. The values of k_d obtained from these transient graphs are plotted against D_c in Figure 24 and apparently show a linear relationship.

The apparent linearity of this plot should not be regarded in any definitive manner since this would require that a plot of D_c vs. q_z (Figure 13) also be linear and pass through the origin. Further data will be required to determine the exact nature of the relationship between D_c and k_d; however, there is convincing evidence that k_d approaches zero as the growth rate diminishes.

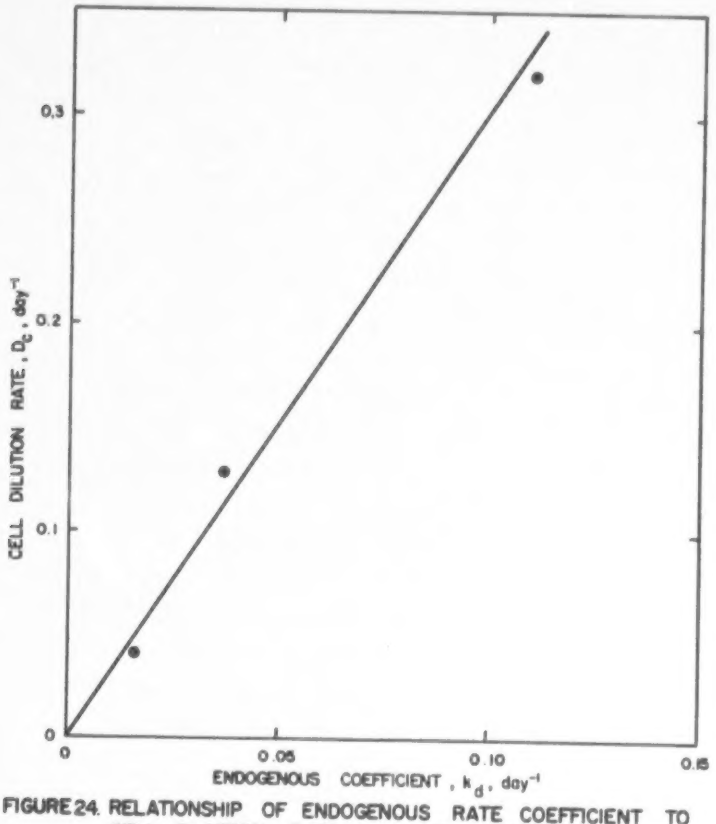


FIGURE 24. RELATIONSHIP OF ENDOGENOUS RATE COEFFICIENT TO CELL DILUTION RATE AS DETERMINED BY TRANSIENT STATE EXPERIMENTS

STEP TRANSIENT EXPERIMENTS

Thirteen laboratory-scale step transient experiments were conducted to determine the behavior of activated sludge systems under rapidly changing environmental conditions. A summary of the initial and short-term asymptotic conditions, as well as the nature of the transient input, is presented in Table XXI while Figures 25 through 37 show the response of effluent soluble COD to the transient as a function of time.

TABLE XXI

SUMMARY OF STEP TRANSIENT EXPERIMENTS

Date - Run	Initial Conditions								Transient Conditions							
	D day^{-1}	$(z_o)_o$ mg/ℓ	$(z_1)_o$ mg/ℓ	$(x_1)_o$ mg/ℓ	$(q_z)_o$ day^{-1}	SVI $\frac{m\ell}{g}$	$\left(\frac{z_1}{z_o}\right)_o$	$\left(\frac{10^3 q_z}{z_o}\right)_o$ $\frac{\ell}{mg\text{-}day}$	D day^{-1}	z_o mg/ℓ	x_1 mg/ℓ	$<z_1>$ mg/ℓ	$<q_z>$ day^{-1}	$\frac{<z_1>}{z_o}$	$\frac{10^3 <q_z>}{z_o}$ $\frac{\ell}{mg\text{-}day}$	Notes
1-03-66-01	3.0	210	21	1170	0.49	340	0.100	2.34	6.00	206	1170	22	0.94	0.107	4.55	D increased
1-10-66-01	3.0	236	17.5	990	0.70	550	0.074	2.96	3.13	620	940	84	1.56	0.135	2.50	z_o increased
1-03-66-02	3.0	207	20	850	0.66	610	0.096	3.20	3.08	404	690	65	1.50	0.161	3.70	z_o increased
1-05-66-03	3.0	610	70	2560	0.80	300	0.115	1.31	3.00	208	2030	23	0.30	0.110	1.45	z_o decreased
1-17-66-05	3.0	414	17	3900	0.31	105	0.042	0.75	3.00	205	3890	10	0.15	0.049	0.73	z_o decreased
1-12-66-06	2.9	214	16	1350	0.43	460	0.075	2.00	5.56	419	1280	34	1.56	0.081	3.73	D and z_o increased
1-18-66-08	3.0	419	20	1820	0.66	540	0.048	1.57	3.35	610	2030	31	1.00	0.051	1.64	z_o increased
5-05-66-20	3.0	52	-	1070	0.12	-	-	-	10.75	200	890	40	2.00	0.200	10.0	D and z_o increased
5-10-66-21	3.0	110	10	2050	0.15	110	0.091	1.37	6.28	113	2050	13	0.31	0.115	2.7	D increased
5-12-66-21	3.0	110	10	2050	0.15	110	0.091	1.37	6.25	213	2170	21	0.55	0.099	2.58	D and z_o increased
5-16-66-21	2.9	313	16	1730	0.56	180	0.051	1.80	5.94	313	1610	19	1.09	0.061	3.48	D increased
5-19-66-22	3.2	210	20	1000	0.57	360	0.095	2.72	9.22	210	1000	30	1.67	0.143	7.95	D increased
5-23-66-24	3.0	212	14	940	0.63	200	0.066	2.95	8.85	212	950	24	1.75	0.113	8.25	D increased

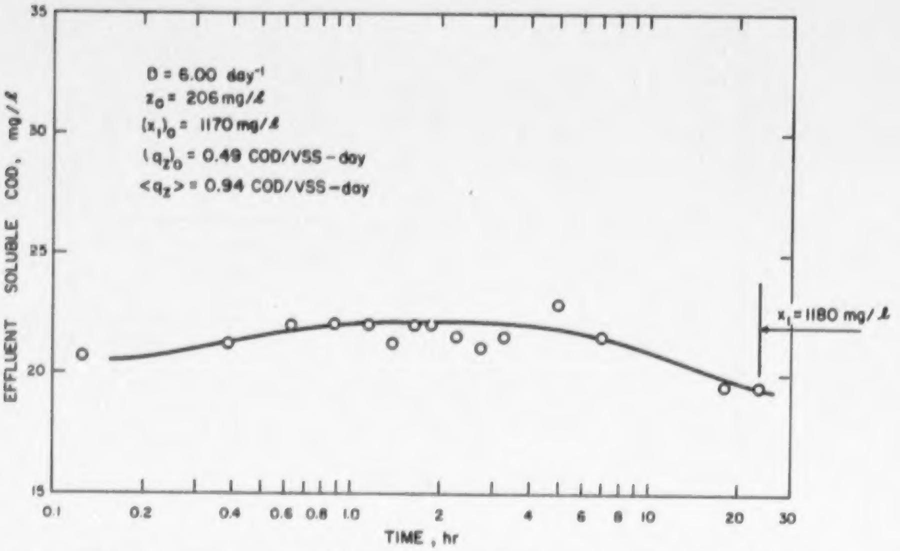


FIGURE 25 . STEP TRANSIENT RESPONSE, RUN I-3-66-01

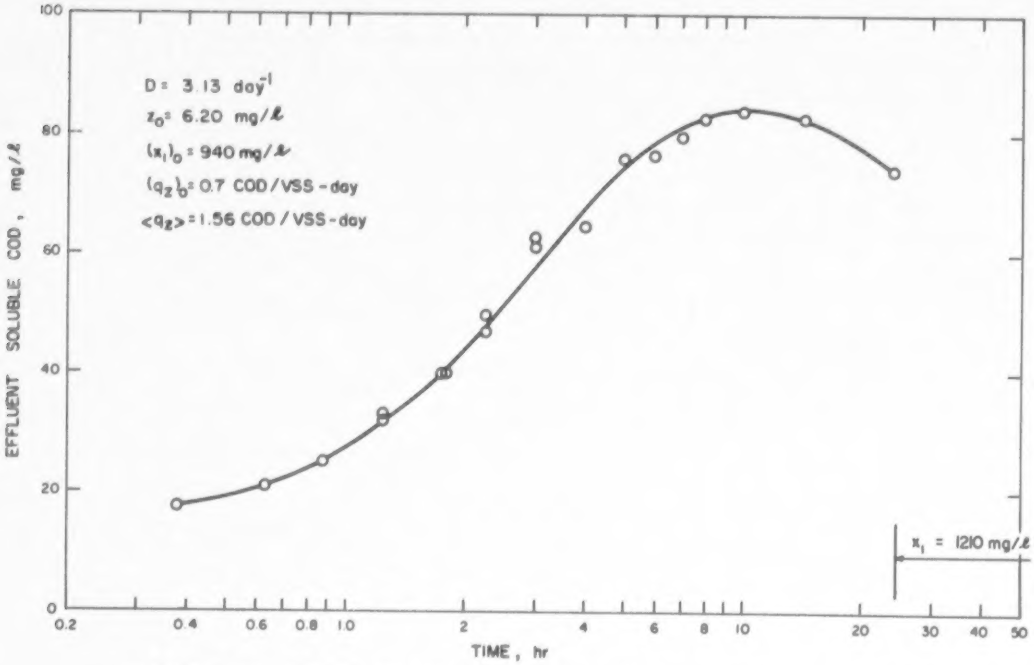


FIGURE 26. STEP TRANSIENT RESPONSE, RUN I-10-66-01

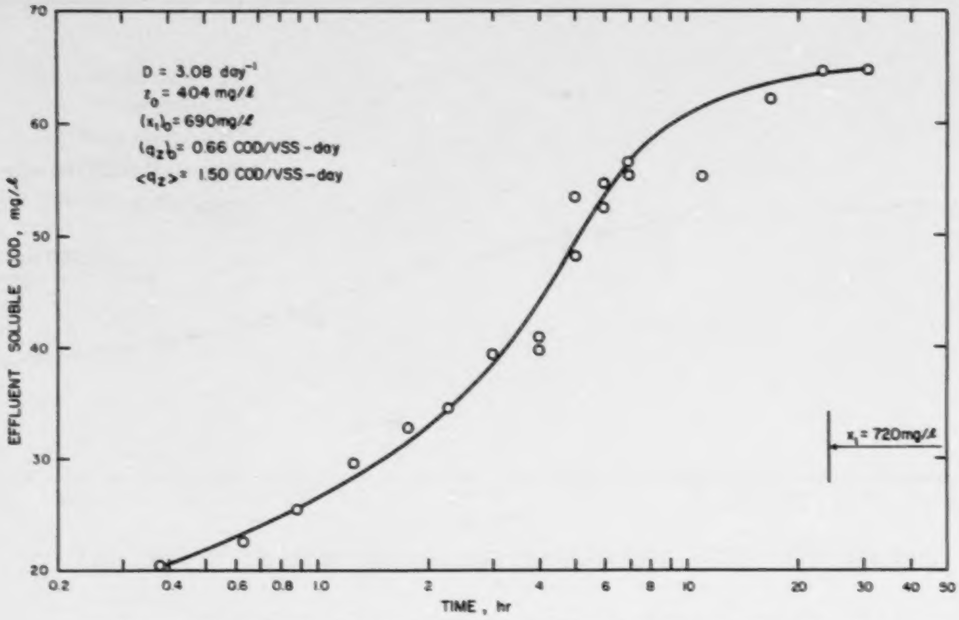


FIGURE 27. STEP TRANSIENT RESPONSE, RUN 01-03-66-02

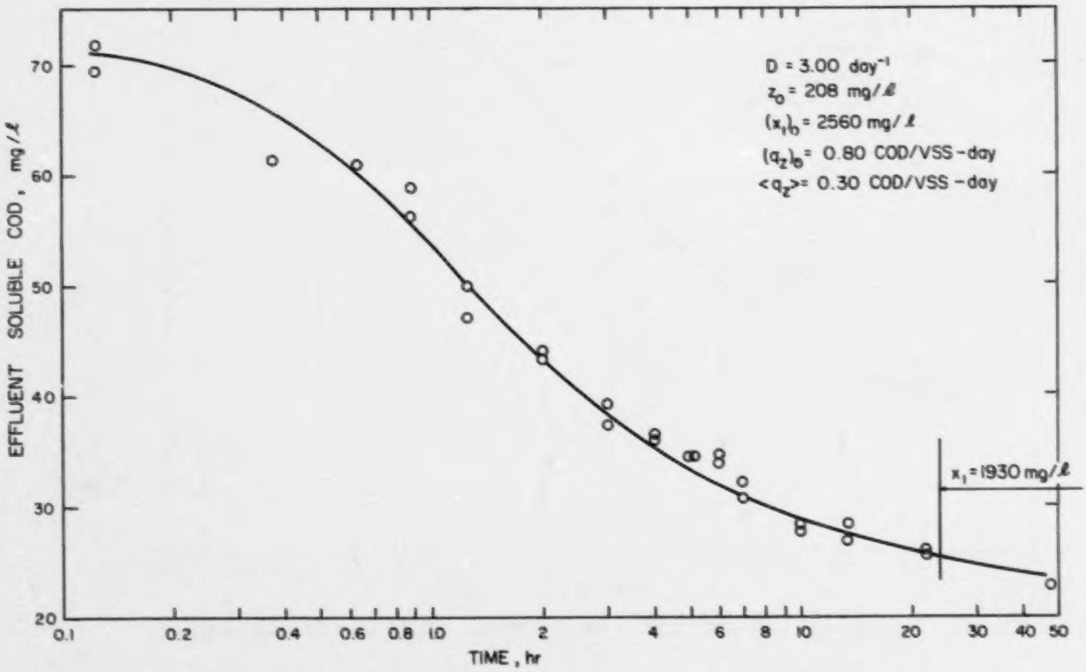


FIGURE 28. STEP TRANSIENT RESPONSE, RUN 01-05-66-03

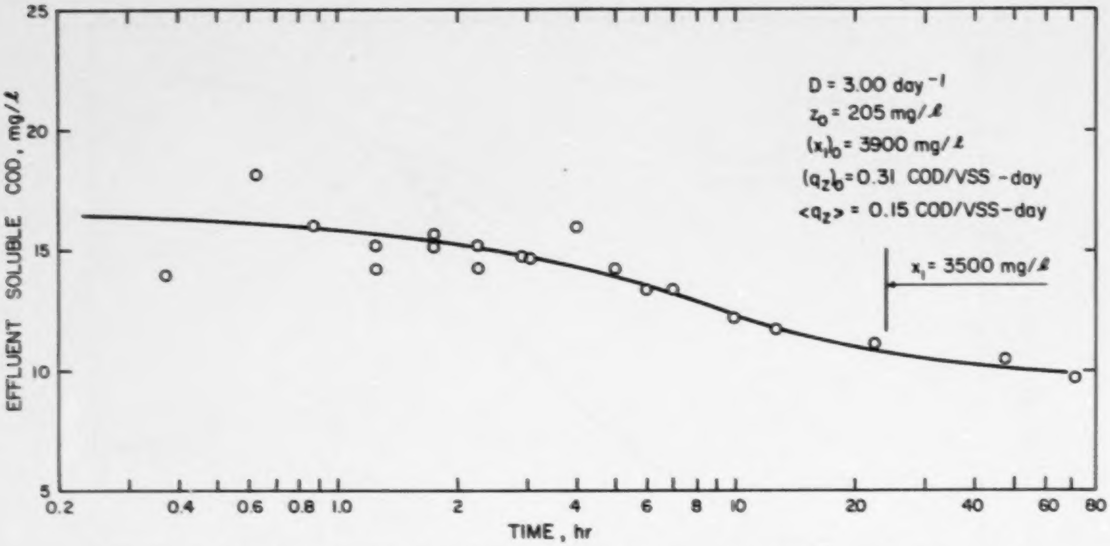


FIGURE 29. STEP TRANSIENT RESPONSE, RUN 01-17-66-05

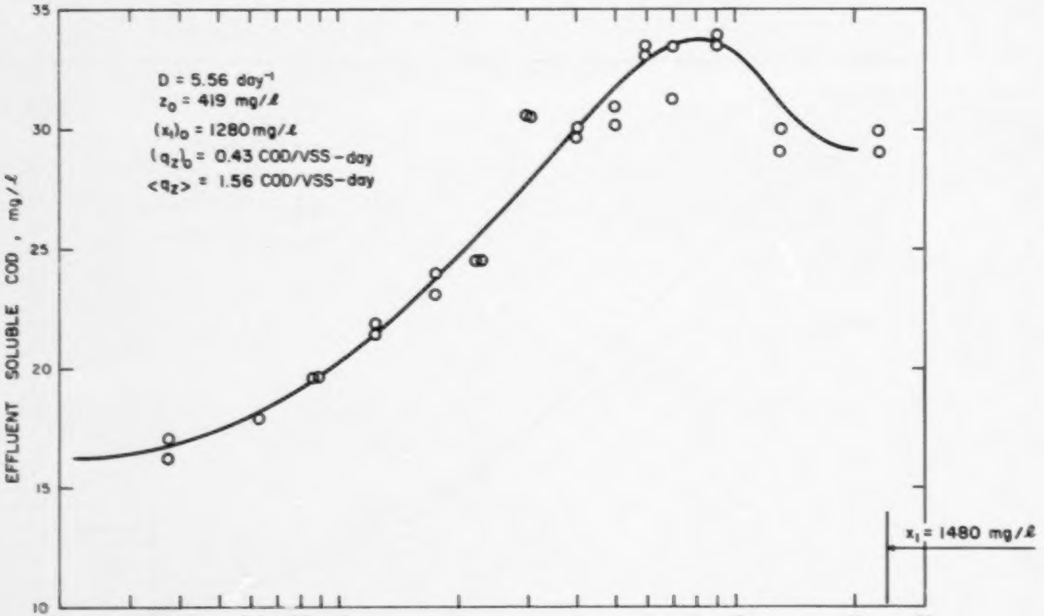


FIGURE 30. STEP TRANSIENT RESPONSE, RUN 01-12-66-06

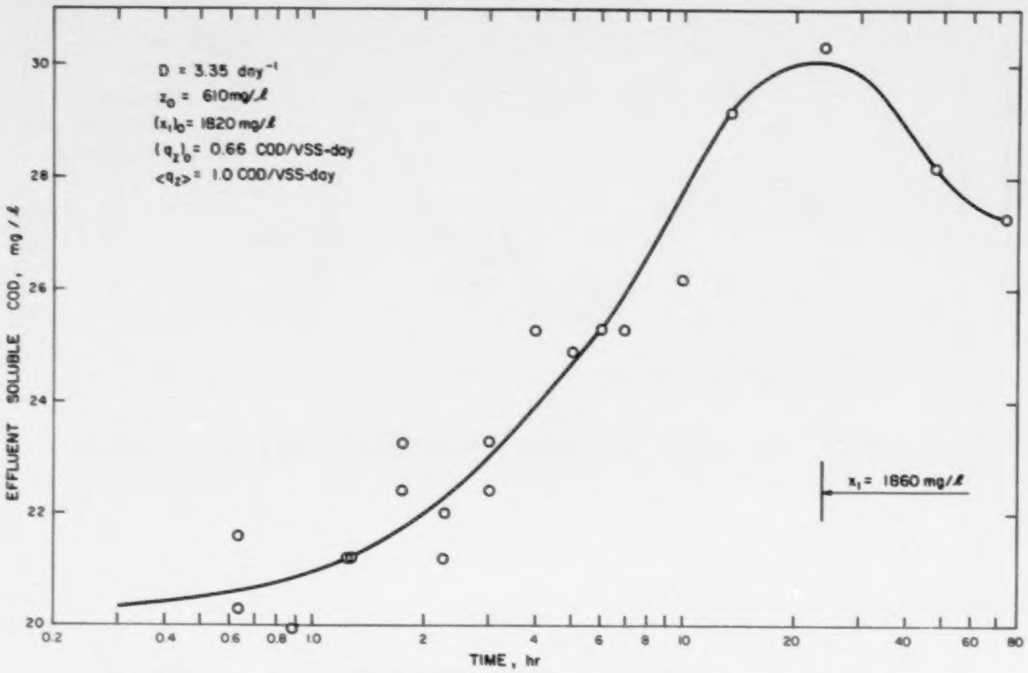


FIGURE 31. STEP TRANSIENT RESPONSE, RUN 01-18-66-08

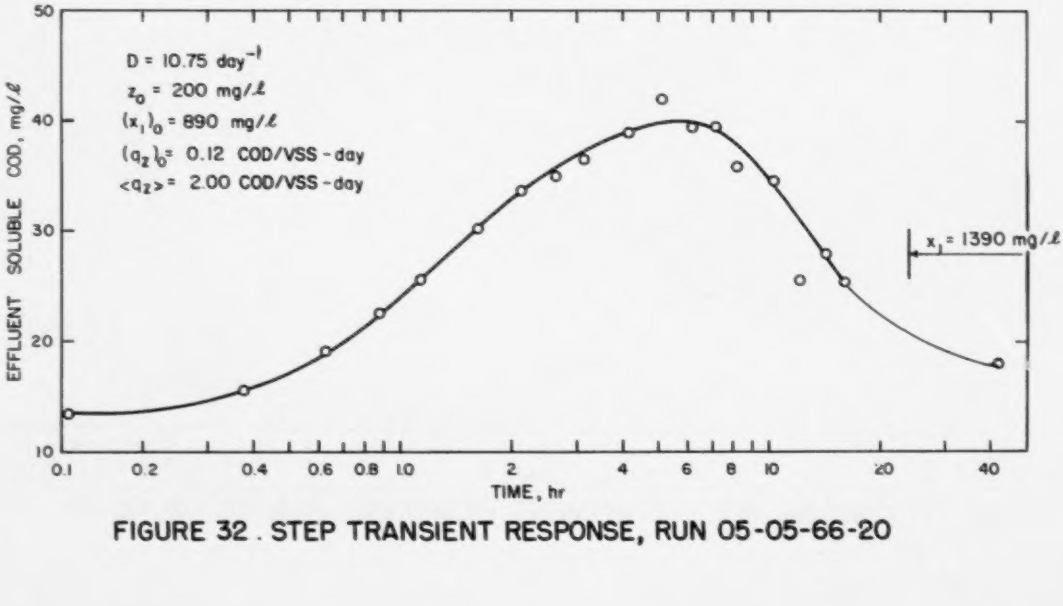


FIGURE 32. STEP TRANSIENT RESPONSE, RUN 05-05-66-20

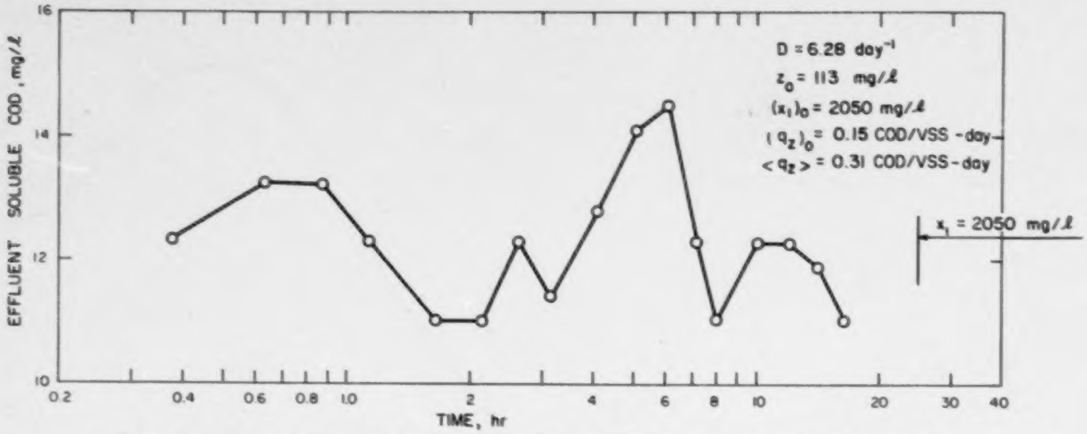


FIGURE 33. STEP TRANSIENT RESPONSE, RUN 05-10-66-21

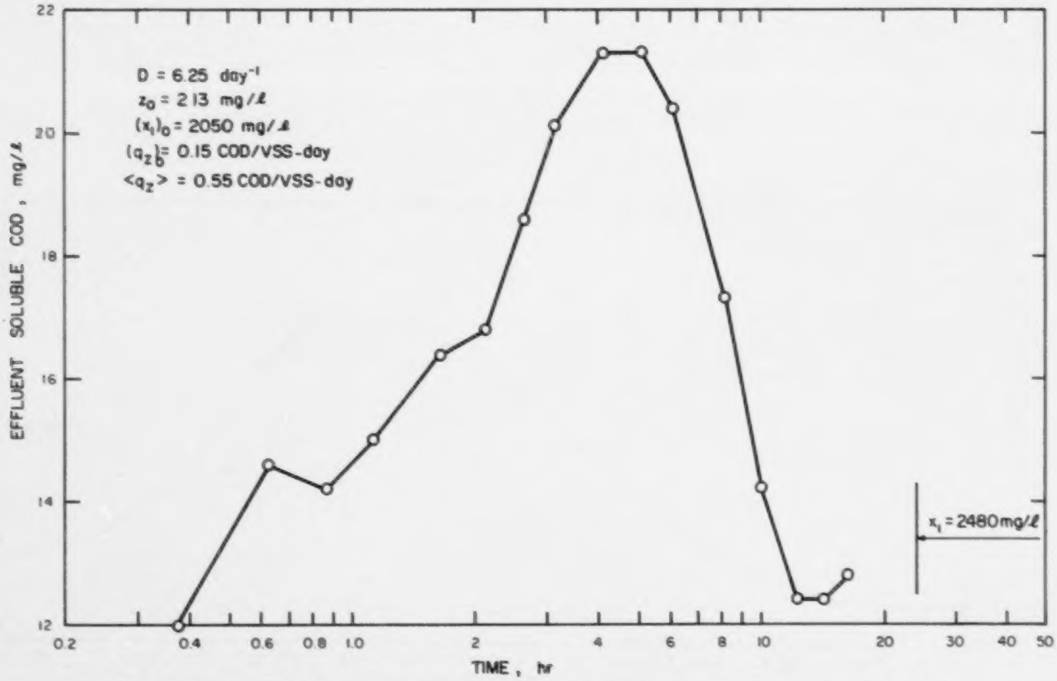


FIGURE 34. STEP TRANSIENT RESPONSE, RUN 05-12-66-21

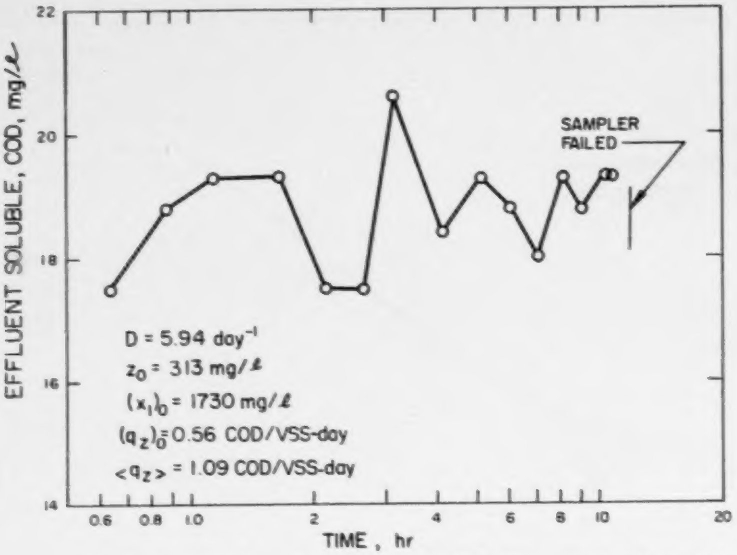


FIGURE 35. STEP TRANSIENT RESPONSE , RUN 05-16-66-21

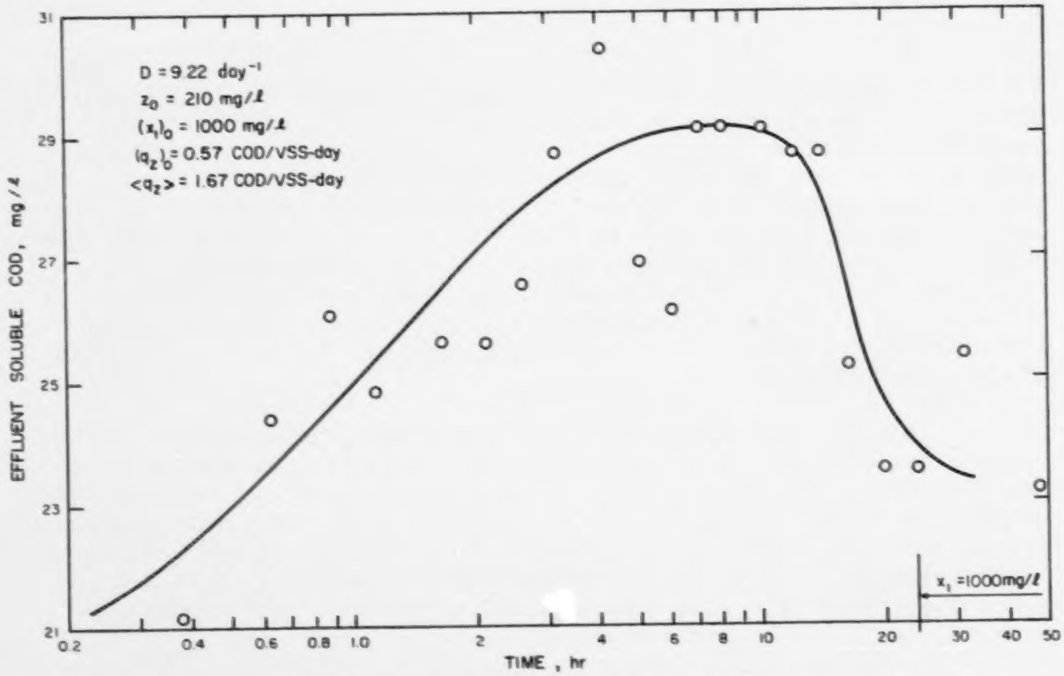


FIGURE 36. STEP TRANSIENT RESPONSE, RUN 05-19-66-22

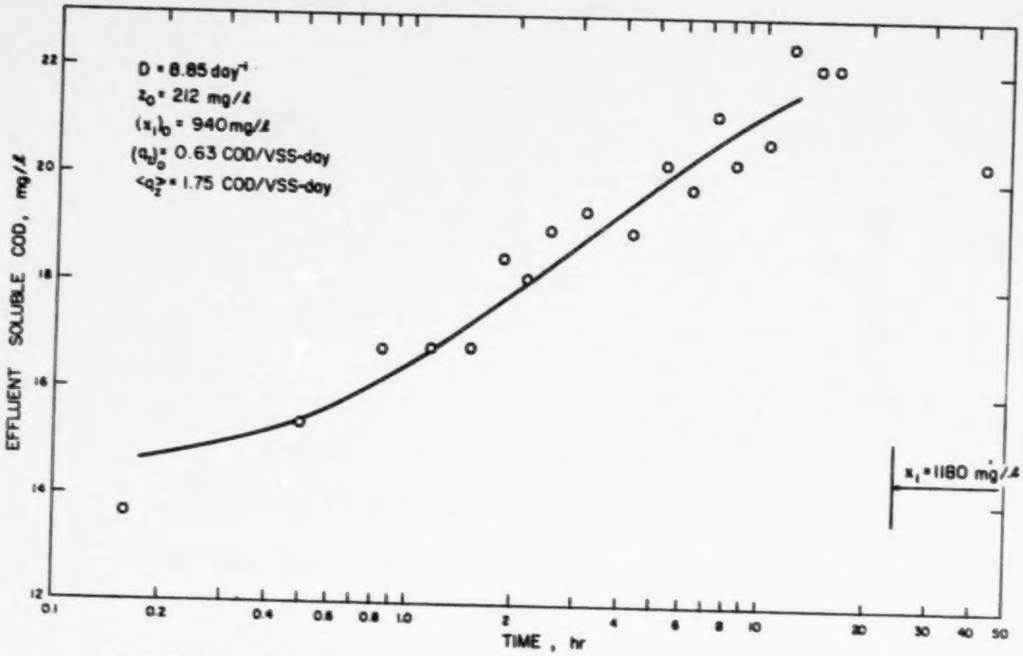


FIGURE 37. STEP TRANSIENT RESPONSE, RUN 05-23-66-24

In Figure 38 the short-term asymptotic values listed in Table XXI are plotted according to Equation 90. According to the theory presented in Chapter III, this plot should be identical to the steady state plot, Figure 14. That is, the fitted slope should equal the overall rate coefficient, k_0, and the intercept should be the limiting ratio of residual soluble COD to feed COD, c_1. However, a two-variable regression analysis of the data gave a slope of 0.066 ℓ/mg-day, an intercept of 0.048, and a correlation coefficient of 0.45. The value of the slope is very low compared with that obtained from the steady state laboratory experiments (0.172 ℓ/mg-day) and the correlation coefficient is unimpressive.

It therefore appears that the steady state and transient state responses of the activated sludge process cannot be predicted with coefficients of the same values. It is possible to argue that: 1) the overall rate coefficient, k_0, is not constant under transient conditions, and 2) that the production of metabolic intermediates or end products under transient conditions is not adequately described by Equation 19. A deviation from this relationship would be caused if intermediate metabolites tended to accumulate.

The latter hypothesis – that end product formation under transient conditions is not predicted accurately by Equation 19 – is favored because it provides an explanation which is also consistent with the response of activated sludge systems to periodic transient inputs, a topic which is considered below.

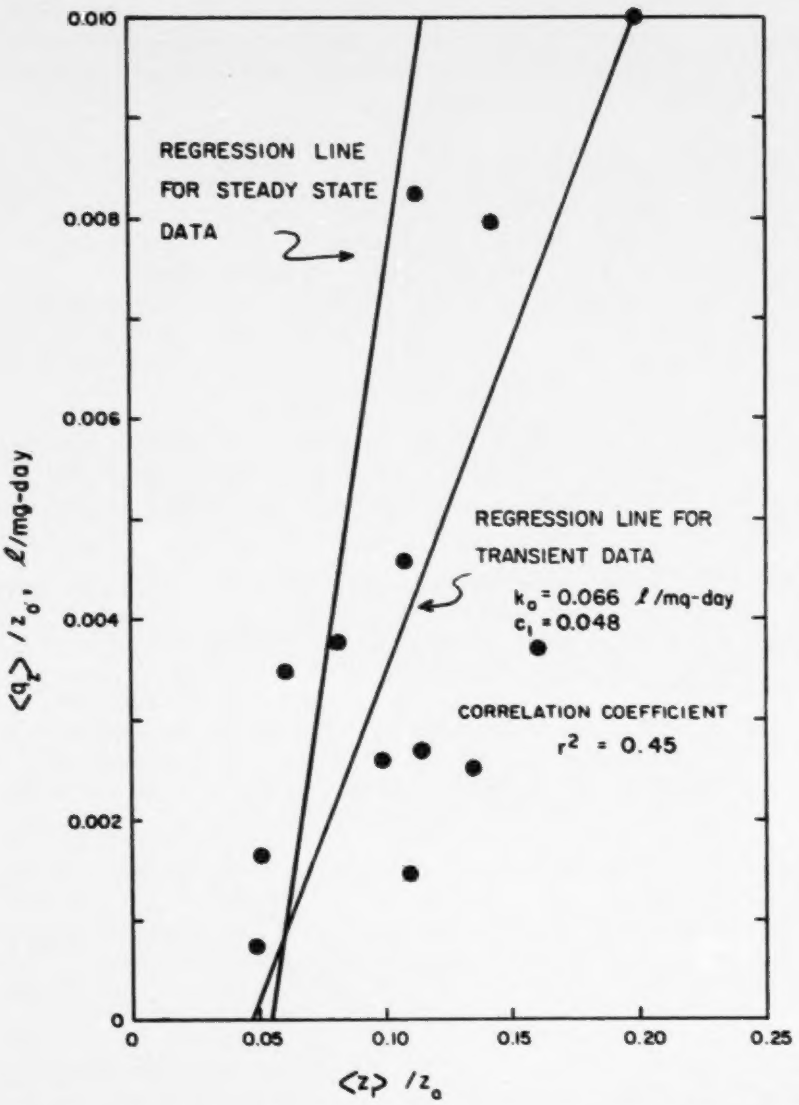


FIGURE 38. KINETIC PLOT OF LABORATORY STEP TRANSIENT RESULTS

PERIODIC TRANSIENT EXPERIMENTS

Two types of periodic input conditions were employed – feed-rate changes and feed-strength variations. Input-output data from experiments in which feed rate was varied periodically are presented in Figures 39 and 40. Figure 41 depicts the results from an experiment in which the feed COD was varied from zero to 388 mg/ℓ and back to zero over a 24-hour period. Theoretical effluent soluble COD responses of the activated sludge systems to periodic transients have been derived in Figures 39, 40, and 41 by using the overall rate coefficient $k_o = 0.172$ ℓ/mg-day obtained from steady state experiments, and by selecting a value of the limiting fractional COD residual, c_1, to give the best correspondence with the laboratory results. Invariably this value of c_1 was higher than the value of 0.056 obtained from laboratory steady state experiments. The values that produced good correspondence with laboratory results were between 0.075 and 0.095.

This modeling of the periodic transient responses of soluble effluent COD was not carried out in a purely arbitrary fashion. The first step in this procedure was to determine whether the values of k_o and c_1 obtained from step transient experiments ($k_o = 0.066$, $c_1 = 0.050$) would adequately describe the experimental results. Reference to Figures 42, 43, and 44 show that these coefficients produced predictions that were inadequate both in magnitude and amplitude. In all cases the effluent soluble COD predicted using these coefficients was too low and showed too great a fluctuation in magnitude.

The use of the overall rate coefficient and limiting fractional residual COD derived from steady state experiments also produced a response that was lower than the experimental results but was of the correct amplitude.

Experimental and predicted responses could be fitted by assuming that the production of nondegradable metabolites (end products or intermediates) was greater under transient than under steady state conditions. This is tantamount to saying that Equation 19 does not describe the production of metabolic end products or intermediates under transient conditions — an assumption that is in agreement with the results obtained by Anderson and Washington [39] whose batch cultures produced intermediate soluble organics which were subsequently metabolized upon depletion of the primary substrate.

Apart from the agreement of predicted and experimentally determined effluent soluble COD responses of activated sludge systems to periodic transients, these data demonstrate that the absolute magnitude of these fluctuations is indeed small. Thus, for a threefold diurnal flow variation at a waste strength of 225 mg/ℓ, a plant operated at a substrate removal rate of 0.17 mg COD/mg VSS-day produced an effluent whose soluble COD fluctuated between 17 and 20 mg/ℓ. A similar flow rate variation at a substrate removal rate of 0.45 mg COD/mg VSS-day produced effluent soluble COD variations of between 20 and 25 mg/ℓ.

Even with severe diurnal variations of feed strength at constant flow rate, the variation of effluent soluble COD was only 10 mg/ℓ. Thus when the feed COD varied from 0 mg/ℓ to 388 mg/ℓ and back to 0 mg/ℓ over a 24-hour period, an activated sludge plant operated at a substrate removal rate of 0.63 mg COD/mg VSS-day produced an effluent soluble COD that varied between 16 and 26 mg/ℓ.

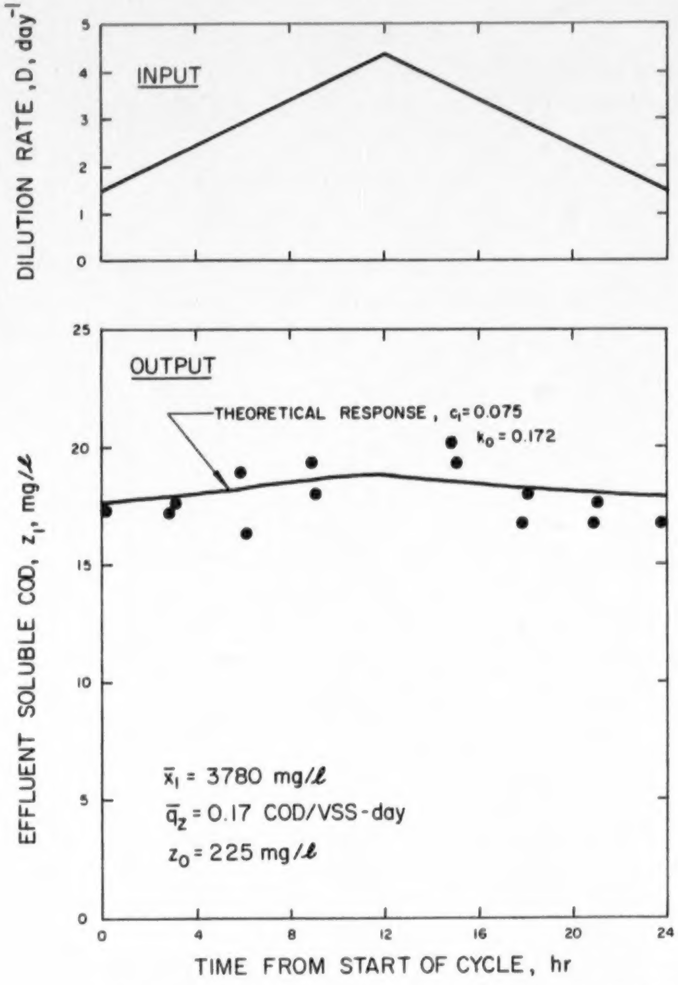


FIGURE 39. PERIODIC TRANSIENT RESPONSE: FLOW RATE VARIATION ONLY, RUN 66-13

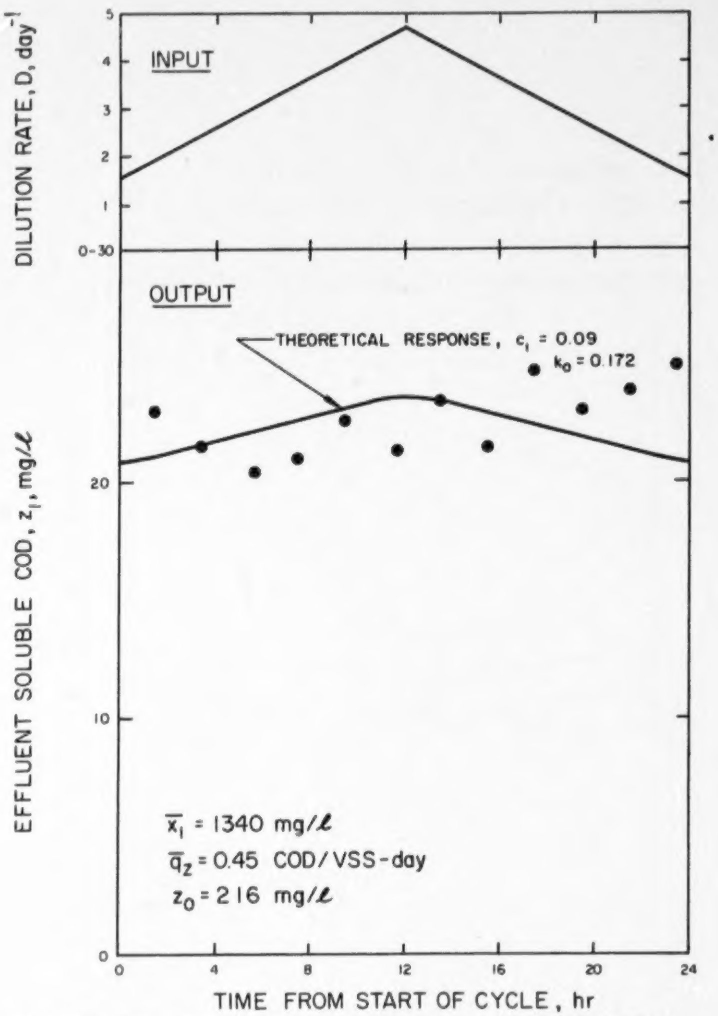


FIGURE 40. PERIODIC TRANSIENT RESPONSE: FLOW RATE VARIATION ONLY, RUN 66-27

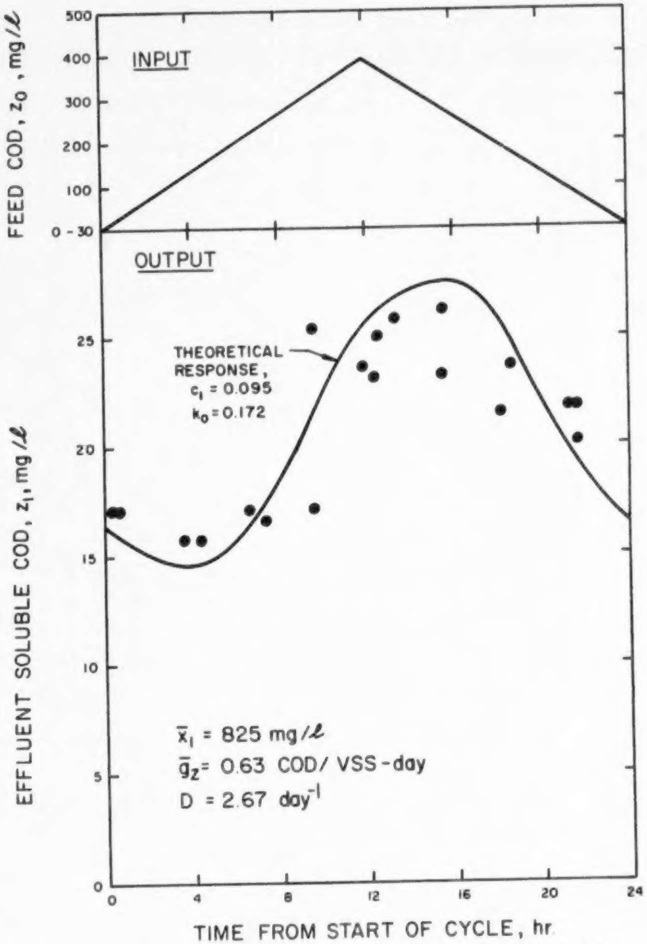


FIGURE 41. PERIODIC TRANSIENT RESPONSE: FEED STRENGTH VARIATION ONLY, RUN 66 - 31

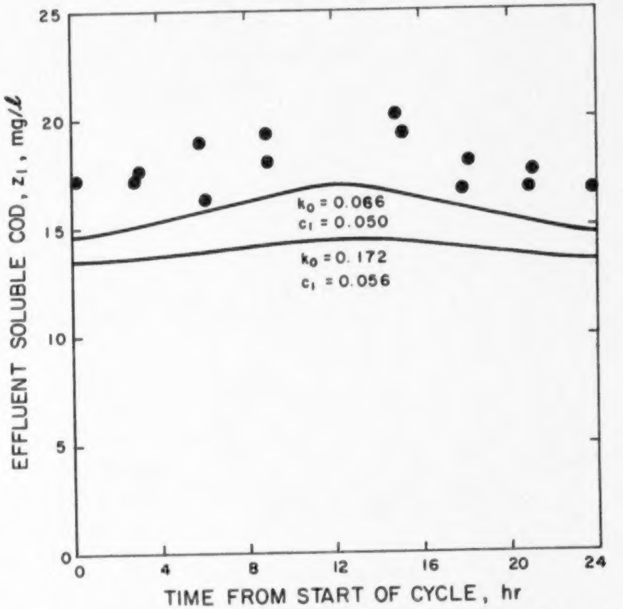


FIGURE 42. COMPARISON OF PREDICTED AND EXPERIMENTAL RESPONSES TO PERIODIC TRANSIENTS: FLOW RATE VARIATION ONLY, RUN 66-13

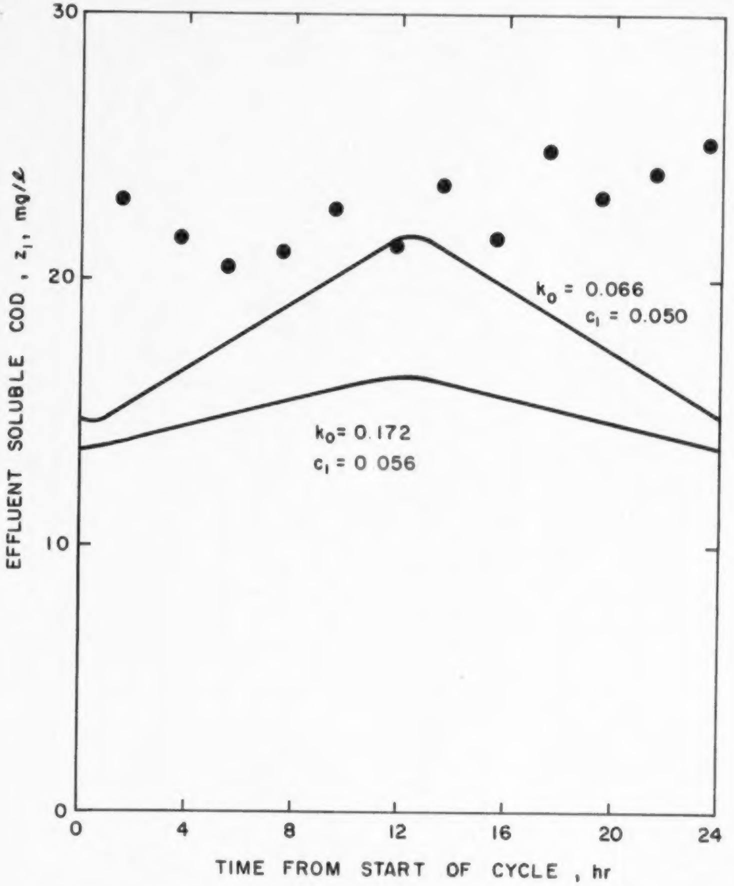


FIGURE 43. COMPARISON OF PREDICTED AND EXPERIMENTAL RESPONSES TO PERIODIC TRANSIENTS: FLOW RATE VARIATION ONLY, RUN 66-27

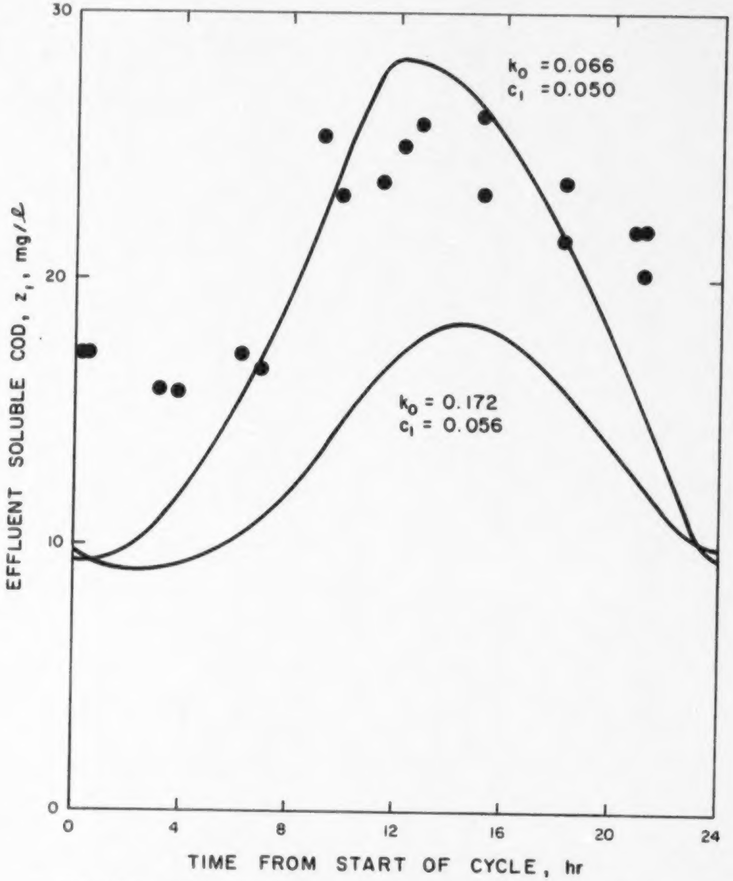


FIGURE 44 . COMPARISON OF PREDICTED AND EXPERIMENTAL RESPONSES TO PERIODIC TRANSIENTS: FEED STRENGTH VARIATION ONLY, RUN 66-31

VI. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

SUMMARY

The overall objective of the research described in this report was to develop a kinetic model of the activated sludge process which was capable of defining both the steady state and transient performance of the process.

Specifically the research:

1. Developed an expression describing the specific growth rate of activated sludge microorganisms as a function of the concentration of the growth-limiting substrate, using the concept of an overall rate coefficient associated with the COD of the reactor system.

2. Developed a model to account for the production of metabolic end products and their accumulation in the process effluent.

3. Developed a kinetic model for the initial uptake of organic substrates by activated sludge microorganisms, and examined the utility of this model and the growth-rate model in describing the transient performance of the activated sludge process.

4. Tested the conclusions drawn from laboratory-scale studies by controlled operation of pilot-scale activated sludge units treating a domestic waste.

Laboratory experiments were conducted at 20° C using a synthetic sewage substrate of which the principal constituents were egg albumen and beef extract. Cell mass was determined as volatile suspended solids and residual growth-limiting substrate was measured as soluble (0.45-μ filterable) COD.

Steady state and transient experiments with pulse, step, and continuously varying (sawtooth function) inputs, were conducted on continuous-flow laboratory units. Batch studies were conducted to evaluate the extent and kinetics of the initial uptake of soluble organic materials by activated sludge.

Laboratory experiments were followed by pilot-scale activated sludge studies using two 562-gal aeration capacity units treating settled domestic sewage.

All experiments included an evaluation of the kinetic parameters of cell yield, Y_z, endogenous rate coefficient, k_d, overall reaction rate coefficient, k_o, and limiting residual COD fraction, c_1. Batch studies were used to evaluate the sorption coefficient, ϵ.

Cell Yield and Endogenous Rate Coefficient

The linear relationship, $D_c = Y_z q_z - k_d$, suggested by several previous investigators was shown to hold for the laboratory experiments conducted in this research. In the range of substrate removal rates from 0.16 to 2.97 mg COD removed/mg VSS-day, a linear regression of a plot of D_c and q_z fitted the data excellently and gave the following values of Y_z and k_d:

$$Y_z = 0.46 \frac{\text{mg VSS}}{\text{mg COD Removed}}$$

$$k_d = 0.08 \text{ day}^{-1} .$$

At low substrate removal rates the points deviated somewhat from the regression line (Figure 13) and were focussed toward the origin. It is believed that this behavior is caused by the variation of k_d with specific growth rate. It was not possible to demonstrate this by steady state experiments because of the interrelationships of Y_z and k_d. However, transient experiments revealed that this was indeed the case for cell dilution rates between 0.04 and 0.32 day^{-1} over which range of conditions k_d varied from 0.016 to 0.11 day^{-1} and was linearly related to D_c.

Pilot-plant data for the treatment of settled domestic sewage showed that for the range of substrate removal rates between 0.18 and 5.47 lb COD removed/lb VSS-day, the yield and endogenous rate coefficients were:

$$Y_z = 0.33 \frac{\text{lb VSS}}{\text{lb COD Removed}}$$

$$k_d = -0.05 \text{ day}^{-1} .$$

It is believed that the negative value of k_d resulted from a systematic error in the sampling program, wherein the sludge in the secondary clarifier was not included in the estimation of the total mass of sludge in the system.

The yield coefficient for activated sludge plants treating domestic sewage in the above range of substrate removal rates agrees well with the coefficient determined in this work. Thus Figure 16 shows data obtained from the SERL pilot plant [23], Whittier Narrows Water Renovation Plant [61], Pomona Water Renovation Plant [24], San Ramon Water Reclamation Plant [63], extended aeration plants at the National Sanitation Foundation [65], and extended aeration plants in Mississippi [64]. From this graph a regression line shows that the Y_z value is 0.336 and k_d is 0.016 day^{-1}. Once again this plot suggests that the points tend toward zero at low values of D_c.

Substrate Residuals and Specific Growth Rate

The Monod relationship for specific growth rate,

$$\mu = \mu_{max} \left(\frac{s_1}{K_s + s_1}\right)$$

or

$$q = q_{max} \left(\frac{s_1}{K_s + s_1}\right)$$

has been widely employed in research on biological waste treatment. Because prior work has shown that K_s is usually much greater than s_1, it was postulated that the Monod equation could be approximated by the expression

$$q = k_o s_1$$

where k_o = an overall rate coefficient. In steady state laboratory experiments, conducted over a range of specific growth rates of 0.087 to 1.53 day^{-1}, k_o was found to be equal to 0.172 ℓ/mg-day.

Residual substrate determinations using soluble measurements were complicated by the fact that soluble metabolic end products were formed by the activated sludge organisms. The separation of the two types of soluble COD was made possible by plotting the experimental data in the form of Equation 74,

$$q_z/z_o = k_o\left(z_1/z_o\right) - c_1 k_o$$

where: c_1 = the limiting residual COD fraction (metabolic end products)

= 0.0556 (laboratory units)

z_o = the feed COD

z_1 = the soluble effluent COD.

In the laboratory systems, the limiting residual COD fraction was shown to be 5.56 percent of the influent COD. Thus in many cases the quantity of metabolic end products significantly exceeded the residual substrate COD — a factor that emphasizes the caution with which residual COD determinations should be applied to the calculation of biological waste treatment efficiency.

The overall rate coefficient obtained from the pilot-plant analyses is significantly greater than that determined in the laboratory-scale experiments. Although deviations from linearity were observed, at loading rates less than 15 lb COD removed/lb VSS-day, the overall rate coefficient was approximately 0.75 ℓ/mg-day and the limiting COD fractional residual was about 15 percent. At higher loading rates the overall rate coefficient increases and, under conditions more typical of normal, activated sludge plant operation COD residuals may fall below 15 percent. The SERL pilot-plant data of Jenkins and Menar [23] and Weddle [60], although obtained at extremely high organic loadings were in excellent agreement with data from activated sludge plants in the field and demonstrated the feasibility of developing high rate activated sludge processes.

Initial Uptake

Experiments were carried out to assess the magnitude of sorptive removal in laboratory systems, which were fed a soluble substrate of egg albumen and beef extract. Theoretical calculations showed that the maximum sorptive capacity of single bacterial cells should be approximately three percent for protein molecules 100 Å in diameter. The laboratory results indicated that assimilative removal took place rapidly since the experimental value of the maximum sorptive capacity was almost five percent. At substrate COD concentrations of less than 100 mg/ℓ, the ratio of specific sorptive removal and residual COD was linear and had a value (the sorption coefficient, ϵ) of 5×10^{-4} ℓ/mg. These results confirm the conclusion of previous workers [26,27,36] that the physical adsorption *per se* of soluble organic constituents by activated sludge organisms is not significant.

Transient Response of the Activated Sludge Process

Thirteen step transient experiments were conducted over a range of substrate removal rates from 0.15 to 2.00 mg COD removed/mg VSS-day. The experimental data were analyzed in accordance with Equation 90, $\langle q_z \rangle / z_o = k_o(\langle z_1 \rangle / z_o) - c_1 k_o$, but the correlation between the parameters $\langle q_z \rangle / z_o$ and $\langle z_1 \rangle / z_o$ was not satisfactory. In most cases the ratio $\langle z_1 \rangle / z_o$ was greater than that predicted on the basis of the overall rate coefficient determined from the steady state experiments.

The periodic transient experiments shed some light on the reasons for the discrepancy. Comparisons of observed with calculated responses of activated sludge systems to periodic transients suggested a higher than predicted concentration of residual metabolic end products or intermediates. The periodic transient runs indicated that the overall rate coefficient under transient conditions was virtually the same as in the steady state.

Moreover, the periodic transient experiments demonstrated that fluctuating flow or fluctuating feed strength had an effect of minor magnitude only on the effluent soluble COD concentration. Thus threefold variations in flow and feed strength variations of from 0 to 400 mg/ℓ COD over a 24-hour period produced 10 mg/ℓ or less change in effluent soluble COD for activated sludge units operating in the conventional range of organic loading.

CONCLUSIONS AND RECOMMENDATIONS

1. The linear expression $D_c = Y_z q_z - k_d$ satisfactorily describes the activated sludge process both in the laboratory and in pilot and field-scale plants treating domestic sewage at substrate removal rates below 5 lb COD removed/lb VSS-day. The substrate removal rate and the yield coefficient in this equation are based on measurements of volatile suspended solids (cells), influent total COD (substrate) and effluent soluble COD. For domestic waste treatment by the activated sludge process in the loading range from 0.1 - 5 lb COD/lb VSS-day it is recommended that a yield coefficient of 0.33 lb VSS/lb COD removed and an endogenous decay constant of < 0.05 day^{-1} be used for design purposes.

2. It was demonstrated by laboratory-scale transient state experiments that the endogenous rate coefficient is not a constant. The coefficient was shown to be a function of the specific growth rate of the sludge. While the magnitude of the coefficient k_d makes variations in its value somewhat insignificant over most of the range of growth rates used practically in the activated sludge process, its absolute magnitude does becomes significant at low growth rates such as those used in the "extended aeration" process modification.

   The decrease of the endogenous decay coefficient with specific growth rate is a plausible explanation for the deviation of the plot of Equation 30 from linearity at low growth rates. It is recommended that further investigations be carried out on the variation of the endogenous decay coefficient with specific growth rate to determine its significance in the operation of activated sludge plants treating domestic wastes. It is possible that the variation of this coefficient (which measures a whole variety of processes leading to a decrease in cell mass) is related to the viability of the activated sludge culture.

3. The use of an overall rate coefficient, based on residual soluble COD, has been justified by the results of this research. Pilot-plant experiments conducted under loading conditions as high as (q =) 42.5 lb COD/lb VSS-day showed that the overall rate coefficient for high-rate activated sludge units treating settled domestic sewage is approximately 0.75 ℓ/mg-day.

   Implied in this conclusion is the fact that loading rates far greater than those currently employed in the activated sludge process can be utilized without substantially increasing the quantity of soluble organic materials in the effluent. However, under higher loading rates, the units studied in this research consistently produced sludges that were difficult to separate by gravity sedimentation.

   It is recommended that:

   a. Further and wider field-scale studies be conducted to obtain a more refined and more widely applicable estimate of the overall rate coefficient, k_o. These studies should be carried out at activated sludge plants serving a variety of communities, so that dependable values for design purposes can be obtained.

b. Methods of cell separation other than gravity sedimentation should be explored since it is this step in the activated sludge process that limits the overall organic matter removal efficiency of the process. If improved methods of cell separation can be developed, it is conceivable that the size of biological waste treatment units can be reduced by an order of magnitude without significant loss in organic matter removal efficiency.

4. This research demonstrated that the effluent soluble COD from the activated sludge process is comprised principally of either metabolic end products or nondegradable organic materials present in the process feed. Pilot-scale and field-scale experiments indicated that a COD of approximately 12 to 15 percent of the feed COD appears in the effluent of an activated sludge unit treating domestic wastes, and that this material cannot be removed by using either longer aeration periods or higher sludge contents. Since the renovation of wastewater demands not only the removal of biologically degradable organic matter but also the removal of "nonbiodegradable organic compounds," especially those that contribute to taste, odor, and color and those that promote growths of aquatic microorganisms, it is recommended that the nature and mode of formation of these residual materials in effluents from biological waste treatment processes be further investigated.

   An investigation of this nature will yield information that will be valuable in determining the choice of tertiary treatment processes for removal of residual organics and should also shed some light on the important biochemical mechanisms in biological waste treatment.

5. Experiments conducted in this research using a "soluble" substrate showed that activated sludge would remove a maximum of only about five percent of its weight of soluble substrate COD by initial uptake. Since some investigators have indicated that substrate, up to 30 percent of the sludge mass, can be removed by this mechanism, it is recommended that further studies should be conducted on the phenomenon of initial uptake to determine its significance in the practical operation of activated sludge treatment plants. It is recommended that these studies should take into account the effect of substrate particle size, sludge specific growth rate, and time allowed for metabolism of substrate after initial uptake.

6. Laboratory experiments on the response of activated sludge units to step, pulse, and periodic transients were conducted using a synthetic sewage. The variation of effluent COD was small under periodic transient conditions of flow and feed strength that were at least as severe as those encountered in the practical operation of systems treating domestic wastes. Predictions of the magnitude of this

response using the theoretical model did not accurately coincide with experimental values when kinetic parameters determined from step transient experiments were used. The steady state overall rate coefficient apparently applies to periodic transient operation; however the production of metabolic end products and intermediates appeared to increase under transient conditions.

In view of the tentative nature of this conclusion it can only be recommended that further work be carried out to substantiate the observations made in this report. It is apparent however that the response of an activated sludge system to transients and the quality of the effluent that it produces under field conditions cannot be accurately represented by steady state experiments. It is therefore recommended that research directed toward the quantification and identification of residual materials in effluents from biological waste treatment plants be carried out on systems which closely duplicate the operating conditions of flow and waste strength variation encountered in the field.

APPENDICES

Appendix	
A	Thermodynamics of Microbial Oxidation and Synthesis in Aerobic Continuous Cultures
B	Theoretical Periodic Variation of Growth-Limiting Substrate
C	Steady State Laboratory Data
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APPENDIX A

THERMODYNAMICS OF MICROBIAL OXIDATION AND SYNTHESIS IN AEROBIC CONTINUOUS CULTURES

The synthesis of cell material is a complex series of biochemical reactions, and consequently it is not presently possible to describe the process in detail. Certain simplifying assumptions are necessary to provide a generalized picture of the process and the energetics involved.

The first assumption is that metabolism of the substrate carbon can be divided into two distinct categories:

(i) Total oxidation of some fraction, a, to provide energy.

(ii) Assimilation of the remaining fraction, b, into cell material.

Assuming that the oxidation state of cell material can be represented by the formula CH_2O[*], the following equation can be written:

$$a \underbrace{\left[CH_\alpha O_\beta\right]}_{\text{substrate}} + a\left(1 + \frac{\alpha}{4} - \frac{\beta}{2}\right)\left[O_2\right] \rightarrow a\left[CO_2\right] + a\left(\frac{\alpha}{2}\right)\left[H_2O\right] \qquad \text{(A-1)}$$

and

$$b\left[CH_\alpha O_\beta\right] + b\left(\frac{\alpha}{4} - \frac{\beta}{2}\right)\left[O_2\right] \rightarrow b\underbrace{\left[CH_2O\right]}_{\text{cell material}} + b\left(\frac{\alpha}{2} - 1\right)\left[H_2O\right]. \qquad \text{(A-2)}$$

Since we deal with continuous cultures, where the reactions represented by Equations A-1 and A-2 take place steadily, the stoichiometric coefficients a and b are rate terms. Thus by dividing by a suitable time increment we obtain:

$$a, b, \ldots = \left(\frac{\text{moles of substrate carbon}}{\text{time}}\right). \qquad \text{(A-3)}$$

Additionally, it is assumed that cells die and lyse in the system. For the time being it will be assumed that the organic materials released by lysis are not metabolized by the surviving cells. This phenomenon can then be written:

$$c\left[CH_2O\right] \rightarrow \text{Decay Products} \qquad \text{(A-4)}$$

where

c = rate of decay of cell material
(moles of cell-carbon/time) .

By summing Equations A-1, A-2 and A-4, the effective cell yields, Y_e, can be obtained,

$$Y_e = \frac{(b - c)(\text{moles of cell carbon synthesized})}{(a + b)(\text{mole of substrate carbon consumed})} . \qquad \text{(A-5)}$$

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[*] This is a matter of convenience and does not affect subsequent developments.

In order to relate the rates a and b, it is necessary to consider the energetics of the processes represented by Equations A-1 and A-2 and to make some assumptions regarding the distribution and efficiency of the material and energy transfer chains within the cell.

The first point to establish is a suitable energy parameter. Since most biological systems operate at very nearly constant temperature and pressure, the Free Energy, F, or more precisely the changes in Free Energy, ΔF, should make a desirable energy parameter. This choice is also favorable because free energy values are tabulated for many of the simpler organic compounds and more importantly because a correlation exists between ΔF and oxygen requirements in the total oxidation reaction. The most serious drawback is the fact that we must deal with non-equilibrium systems which introduce a type of "frictional resistance" to the energy transfer chain. Hence the free energy change in Equation A-1, which will be designated the free energy (change) of oxidation for substrate, $\left(\Delta F_{ox}\right)_{sub}$, cannot be entirely transferred to energy-rich compounds in the cell such as ATP, UTP, etc. Thus in the glycolysis and Krebs cycles the efficiency of energy capture is about 40 percent. For this system the energy captured is:

$$E_1 = -\lambda_1 \left(\Delta F_{ox}\right)_{sub} \text{ (k-cal/mole of substrate carbon)} \qquad \text{(A-6)}$$

where λ_1 = fractional energy capture by energy-rich compounds.

Using arguments similar to those presented above, it can be shown that an amount of energy, E_2, in excess of the stoichiometric quantity is required to drive the synthetic reaction:

$$E_2 = \left[\lambda_2 + \lambda_3 \left(\Delta F_{syn}\right)\right] \text{ (k-cal/mole of substrate carbon)} \qquad \text{(A-7)}$$

where λ_2, λ_3 characterize the energy transfer efficiency of the synthesis reaction scheme; and $\left(\Delta F_{syn}\right)$ = the free energy change for reaction A-2 per mole of substrate carbon.

It is assumed that the other energy requirements of the cell can be combined into one term, which will be designated the energy of maintenance. Let E_m = the rate of utilization of energy for cell maintenance per mole of cell carbon (k-cal/mole of cell carbon/time.

The energy fluxes associated with these rates of energy use are:

$$N \cdot E_m \text{ (k-cal/time)} \qquad \text{(A-8)}$$

where N = total number of moles of cell carbon in the "system"

$$a\,E_1 = -\,a\,\lambda_1 \left(\Delta F_{ox}\right)_{sub} = \text{rate capture of energy in high energy compounds (k-cal/time)} \qquad \text{(A-9)}$$

$$b\,E_2 = b\,\lambda_2 + b\,\lambda_3 \left(\Delta F_{syn}\right) = \text{rate of energy utilization for synthesis of cell material (k-cal/time)}\,. \qquad \text{(A-10)}$$

The rates can be equated as follows:

$$\begin{aligned} a\,E_1 = b\,E_2 + N\,E_m &- a\,\lambda_1 \left(\Delta F_{ox}\right)_{sub} \\ &= b\,\lambda_2 + b\,\lambda_3 \left(\Delta F_{syn}\right) + N\,E_m \\ &= b\left[\lambda_2 + \lambda_3 \left(\Delta F_{syn}\right) + \frac{N}{b} \cdot E_m\right]. \end{aligned} \qquad \text{(A-11)}$$

However, N is the total number of moles of cell carbon in the system and b is the rate at which cell carbon is being synthesized. Therefore:

$$\frac{b}{N} \equiv \frac{1}{x}\frac{dx}{dt} = \mu, \text{ the specific growth rate} \qquad \text{(A-12)}$$

and Equation A-11 can be written

$$- a\,\lambda_1 \left(\Delta F_{ox}\right)_{sub} = b\left[\lambda_2 + \lambda_3\left(\Delta F_{syn}\right) + \frac{E_m}{\mu}\right]. \qquad \text{(A-13)}$$

The relationship between a and b can now be taken from Equation A-13 and applied to A-5

$$a = b\left\{\frac{\lambda_2 + \lambda_3\left(\Delta F_{syn}\right) + E_m/\mu}{-\lambda_1\left(\Delta F_{ox}\right)_{sub}}\right\}. \qquad \text{(A-14)}$$

Hence

$$Y_e = \frac{-\left(1 - c/b\right)\lambda_1\left(\Delta F_{ox}\right)_{sub}}{-\lambda_1\left(\Delta F_{ox}\right)_{sub} + \lambda_2 + \lambda_3\left(\Delta F_{syn}\right) + \frac{E_m}{\mu}}. \qquad \text{(A-15)}$$

Now

$$\frac{c}{b} = \frac{\text{decay rate}}{\text{gross growth rate}} = \frac{k_D}{\mu} \qquad \text{(A-16)}$$

where k_D = decay rate coefficient.

Substituting Equation A-16 into A-15 and inverting the latter gives:

$$\frac{1}{Y_e} = \left[\frac{\lambda_2 + \lambda_3\left(\Delta F_{syn}\right) - \lambda_1\left(\Delta F_{ox}\right)_{sub}}{-\left(1 - \frac{k_D}{\mu}\right)\lambda_1\left(\Delta F_{ox}\right)_{sub}}\right] + \left[\frac{E_m}{-\lambda_1\left(\Delta F_{ox}\right)_{sub}\left(\mu - k_D\right)}\right]. \qquad \text{(A-17)}$$

If all the factors in the first term which do not involve μ are lumped together, they represent the true or theoretical yield, Y_o.

$$\frac{1}{Y_o} = \frac{\lambda_2 + \lambda_3\left(\Delta F_{syn}\right) - \lambda_1\left(\Delta F_{ox}\right)_{sub}}{-\lambda_1\left(\Delta F_{ox}\right)_{sub}}\left(\frac{\text{moles of substrate carbon}}{\text{mole of cell carbon}}\right). \qquad \text{(A-18)}$$

Then Equation A-17 becomes

$$\frac{1}{Y_e} = \frac{\mu}{Y_o\left(\mu - k_D\right)} + \frac{E_m}{-\lambda_1\left(\Delta F_{ox}\right)_{sub}\left(\mu - k_D\right)}. \qquad \text{(A-19)}$$

However, the net growth rate $\left(\mu - k_D\right)$ must equal the specific rate of cells issuing from the system or the reciprocal mean cell residence period, $1/\theta_c$.

Then

$$\frac{1}{Y_e} = \frac{\mu\ \theta_c}{Y_o} + \frac{E_m\ \theta_c}{-\ \lambda_1\ \left(\Delta F_{ox}\right)_{sub}}\ . \qquad (A\text{-}20)$$

In addition, the rate of endogenous metabolism can be written in terms of the amount of cell material that would have been formed in the absence of endogenous metabolism, as follows:

$$k_e = \frac{Y_o\ E_m}{-\ \lambda_1\ \left(\Delta F_{ox}\right)_{sub}}\ . \qquad (A\text{-}21)$$

Then

$$\frac{1}{Y_e} = \frac{1}{Y_o}\left[\mu\ \theta_c + k_e\theta_c\right]\ . \qquad (A\text{-}22)$$

Again using

$$\frac{1}{\theta_c} = \mu - k_D$$

$$\frac{1}{Y_e} = \frac{1}{Y_o}\left[1 + \left(k_D + k_e\right)\theta_c\right]\ . \qquad (A\text{-}23)$$

Defining k_d as the endogenous rate coefficient

$$k_d = k_D + k_e$$

$$\frac{1}{Y_e} = \frac{1}{Y_o}\left(1 + k_d\theta_c\right)\ . \qquad (A\text{-}24)$$

However, from Equation A-5

$$Y_e = \frac{x_2}{\left(s_o - s_1\right)} \qquad (A\text{-}25)$$

where: x_2 = cell carbon lost from the system by discharge in process effluent, $[ML^{-3}]$

s_o = substrate carbon in process feed, $[ML^{-3}]$

s_1 = substrate carbon in process effluent, $[ML^{-3}]$.

Also by previous definition

$$x_2 = \left(\frac{\theta}{\theta_c}\right) x_1 \qquad (A\text{-}26)$$

$$\therefore\quad Y_e = \frac{\theta}{\theta_c}\left(\frac{x_1}{s_o - s_1}\right)\ . \qquad (A\text{-}27)$$

But

$$\frac{s_o - s_1}{\theta x_1} = q\text{, the specific substrate removal rate.} \qquad \text{(A-28)}$$

Therefore

$$Y_e = \frac{1}{\theta_c q} . \qquad \text{(A-29)}$$

Hence Equation A-24 becomes:

$$\theta_c q = \frac{1}{Y_o}\left(1 + k_d \theta_c\right)$$

$$Y_o q = \frac{1}{\theta_c} + k_d \qquad \text{(A-30)}$$

which yields the steady state biomass conservation equation,

$$\boxed{\frac{1}{\theta_c} = Y_o q - k_d.} \qquad (30)$$

On the basis of the above, the following conclusions can be stated:

1. Y_o is the true yield coefficient which can never be attained since μ must be infinite, as shown in Equation A-19:

$$\lim_{\mu \to \infty} \frac{1}{Y_e} = \frac{1}{Y_o} + 0 .$$

2. Since k_d is associated with cell mass which does not appear in the process effluent, the true nature of the phenomenon cannot be determined without resorting to special techniques.

RELATIONSHIP OF THEORETICAL YIELD COEFFICIENT TO SUBSTRATE COD

Inverting Equation A-18 gives

$$Y_o = \frac{-\lambda_1 \left(\Delta F_{ox}\right)_{sub}}{-\lambda_1 \left(\Delta F_{ox}\right)_{sub} + \lambda_2 + \lambda_3 \left(\Delta F_{syn}\right)} . \qquad \text{(A-31)}$$

In order to relate $\left(\Delta F_{ox}\right)_{sub}$ to $\left(\Delta F_{syn}\right)$, it is necessary to make the following energy balances, using the free energy of formation $\left(\Delta F_f\right)$ for certain of the reactants and products:

Synthesis of Cell Material (C.M.)

$$\left[CH_\alpha\ O_\beta\right] + \left(\frac{\alpha}{4} - \frac{\beta}{2}\right)\left[O_2\right] \rightarrow \left[CH_2O\right] + \left(\frac{\alpha}{2} - 1\right)\left[H_2O\right] \qquad \text{(A-32)}$$

$$\left(\Delta F_{syn}\right) = \left(\Delta F_f\right)_{C.M.} + \left(\frac{\alpha}{2} - 1\right)\left(\Delta F_f\right)_{H_2O} - \left(\Delta F_f\right)_{sub} . \qquad \text{(A-33)}$$

Total Oxidation of Substrate

$$\left[CH_{\alpha}\ O_{\beta}\right] + \left(1 + \frac{\alpha}{4} - \frac{\beta}{2}\right)\left[O_2\right] \rightarrow \left[CO_2\right] + \left(\frac{\alpha}{2}\right)\left[H_2O\right] \qquad \text{(A-34)}$$

$$\left(\Delta F_{ox}\right)_{sub} = \left(\Delta F_f\right)_{CO_2} + \left(\frac{\alpha}{2}\right)\left(\Delta F_f\right)_{H_2O} - \left(\Delta F_f\right)_{sub} \qquad \text{(A-35)}$$

Total Oxidation of Cell Material

$$\left[CH_2O\right] + \left[O_2\right] \rightarrow \left[CO_2\right] + \left[H_2O\right] \qquad \text{(A-36)}$$

$$\left(\Delta F_{ox}\right)_{C.M.} = \left(\Delta F_f\right)_{CO_2} + \left(\Delta F_f\right)_{H_2O} - \left(\Delta F_f\right)_{C.M.} \ . \qquad \text{(A-37)}$$

From Equation A-34

$$\left(\Delta F_f\right)_{sub} = \left(\Delta F_f\right)_{CO_2} + \left(\frac{\alpha}{2}\right)\left(\Delta F_f\right)_{H_2O} - \left(\Delta F_{ox}\right)_{sub} \ . \qquad \text{(A-38)}$$

From Equation A-37

$$\left(\Delta F_f\right)_{C.M.} = \left(\Delta F_f\right)_{CO_2} + \left(\Delta F_f\right)_{H_2O} - \left(\Delta F_{ox}\right)_{C.M.} \ . \qquad \text{(A-39)}$$

Subtracting Equation A-38 from A-39,

$$\left(\Delta F_f\right)_{C.M.} - \left(\Delta F_f\right)_{sub} = -\left(\frac{\alpha}{2} - 1\right)\left(\Delta F_f\right)_{H_2O} + \left(\Delta F_{ox}\right)_{sub} - \left(\Delta F_{ox}\right)_{C.M.} \ . \qquad \text{(A-40)}$$

Substitution of Equation A-40 in A-33 yields

$$\boxed{\left(\Delta F_{syn}\right) = \left(\Delta F_{ox}\right)_{sub} - \left(\Delta F_{ox}\right)_{C.M.} \ .} \qquad \text{(A-41)}$$

Hence Equation A-31 becomes

$$Y_o = \frac{-\lambda_1\left(\Delta F_{ox}\right)_{sub}}{-\lambda_1\left(\Delta F_{ox}\right)_{sub} + \lambda_3\left(\Delta F_{ox}\right)_{sub} - \lambda_3\left(\Delta F_{ox}\right)_{C.M.} + \lambda_2} \qquad \text{(A-42)}$$

where $Y_o = \dfrac{\text{moles of cell carbon}}{\text{mole of substrate carbon}}$.

Servizi and Bogan [17] have shown

$$\left(\Delta F_{ox}\right)_{sub} \simeq \left(-\ 110\ \frac{\text{k-cal}}{\text{mole } O_2}\right)\left(COD\right)_{sub} \ . \qquad \text{(A-43)}$$

Thus

$$Y_o \simeq \frac{\lambda_1\left(COD\right)_{sub}}{\left(\lambda_1 - \lambda_3\right)\left(COD\right)_{sub} + \lambda_3\left(COD\right)_{C.M.} + \lambda_2'} \qquad \text{(A-44)}$$

where

$$\lambda_2' = \left(110 \, \frac{\text{moles } O_2}{\text{k-cal}}\right) \lambda_2 \, .$$

For a given substrate it can be assumed as a first approximation that $(\text{COD})_{\text{C.M.}}$ is a constant.

Then

$$\boxed{Y_o' = \frac{\lambda_1}{(\lambda_1 - \lambda_3)\,(\text{COD})_{\text{sub}} + \lambda_4}} \qquad (\text{A-45})$$

where $Y_o' = \dfrac{\text{moles of cell carbon}}{\text{mole of substrate COD}}$

$\lambda_4 = \lambda_3 \, (\text{COD})_{\text{C.M.}} + \lambda_2'$

where $\lambda_3 \, (\text{COD})_{\text{C.M.}}$ is an approximate constant.

Thus the theoretical yield based on the COD of the substrate is not a true constant. However, depending on the relative values of λ_1 and λ_3, it might be nearly so. There is very little experimental evidence available, but it might be presumed that the energy transfer efficiencies represented by these two factors are nearly the same. If this is the case, then the COD-based yield coefficient takes on more significance as a fundamental constant.

APPENDIX B

PERIODIC VARIATION OF GROWTH-LIMITING SUBSTRATE

Beginning with Equation 81,

$$\frac{ds_1}{dt} + \left(\frac{k_o x_1 + D}{1 + \epsilon x_1}\right) s_1 = \frac{Dz_o}{1 + \epsilon x_1} \tag{B-1}$$

and making the definitions expressed in Equations 82-84, one obtains Equation 85,

$$\frac{ds_1}{dt} + \lambda s_1 = \lambda_5 + \frac{\lambda_6}{c} [H(c,t)] \ . \tag{B-2}$$

The first term on the right contributes the following to the long-term solution for s_1:

$$\lim_{t \to \infty} f_1(t) = \frac{\lambda_5}{\lambda} \ . \tag{B-3}$$

To obtain a solution for the second term, each period of the sawtooth function must be considered separately. At the end of the n^{th} period, let

$$t = 2(n + 1)c \ . \tag{B-4}$$

Then, the sawtooth function is mathematically defined as follows:

$$\begin{array}{ll} 2nc \leqq t \leqq (2n+1)c & H(c,t) = t - 2nc \\ (2n+1)c \leqq t \leqq (2n+2)c & H(c,t) = (2n+2)c - t \ . \end{array} \tag{B-5}$$

If this second solution is designated

$$s_1 = \left(\frac{\lambda_6}{c}\right) f_2(t) \ , \tag{B-6}$$

Then, for large values of t

$$f_2(t) = e^{-\lambda t} \int_0^t [H(c,t)] \, e^{\lambda t} \, dt \tag{B-7}$$

or

$$f_2(t) = \sum_{n=0}^{n=k} e^{-\lambda t} \left\{ \int_{2nc}^{(2n+1)c} (t - 2nc)\, e^{\lambda t}\, dt + \int_{(2n+1)c}^{(2n+2)c} [(2n+2)c - t]\, e^{\lambda t}\, dt \right\}$$

$$+ e^{-\lambda t} \int_{(2k+2)c}^{t} e^{\lambda t}\, [H(c,t)]\, dt \, . \tag{B-8}$$

Carrying out the integration under the Σ gives

$$e^{-\lambda t} \sum_{n=0}^{k} \left\{ \left[\frac{e^{\lambda t}}{\lambda^2} (\lambda t - 1) - (2nc) \frac{e^{\lambda t}}{\lambda} \right]_{2nc}^{(2n+1)c} \right.$$

$$\left. + \left[(2n + 2)c \frac{e^{\lambda t}}{\lambda} - \frac{e^{\lambda t}}{\lambda^2} (\lambda t - 1) \right]_{(2n+1)c}^{(2n+2)c} \right\} . \tag{B-9}$$

Inserting the limits yields

$$\left[\frac{1 - e^{\lambda c}}{\lambda} \right]^2 \sum_{n=0}^{k} e^{\lambda t}\, e^{n(2\lambda c)} \, . \tag{B-10}$$

However, the finite series in Equation B-10 can be evaluated

$$\sum_{n=0}^{k} e^{n(2\lambda c)} = \left[\frac{\left(e^{2\lambda c}\right)^{k+1} - 1}{e^{2\lambda c} - 1} \right] . \tag{B-11}$$

Thus Equation B-10 becomes

$$\left[\frac{1 - e^{\lambda c}}{\lambda} \right]^2 \left[\frac{e^{-\lambda[t-(2k+2)c]} - e^{-\lambda t}}{e^{2\lambda c} - 1} \right] .$$

At this point, it is convenient to adopt a new time datum. Let T = time elapsed from beginning of $(k + 1)^{th}$ period.

Thus

$$T = t - (2k + 2)c \, . \tag{B-12}$$

with $0 \leq T \leq 2c$.

Also, let k be large, so that t is also large, and so

$$e^{-\lambda t} \rightarrow 0 \; , \tag{B-13}$$

then

$$f_2(t) = \frac{\left(e^{\lambda c} - 1\right)}{\left(\lambda^2 \left(e^{\lambda c} + 1\right)\right)} e^{-\lambda T} + e^{-\lambda t} \int_{(2k+2)c}^{t} e^{\lambda t} \left[H(c,t)\right] dt \tag{B-14}$$

$$= \frac{1}{\lambda^2} \tanh\left(\frac{\lambda c}{2}\right) e^{-\lambda T} + e^{-\lambda t} \int_{(2k+2)c}^{t} e^{\lambda t} \left[H(c,t)\right] dt \; . \tag{B-15}$$

Making the substitution (Equation B-12), the last term of Equation B-15 becomes

$$e^{-\lambda[(2k+2)c+T]} \int_{(2k+2)c}^{(2k+2)c+T} e^{\lambda t} \left[H(c,t)\right] dt \tag{B-16}$$

and must be evaluated for each half of the (k + 1)th period individually. The solutions are:

(1) For $0 \leqq T \leqq c$

$$\frac{\lambda T - 1}{\lambda^2} + \frac{e^{-\lambda T}}{\lambda^2} \; ; \tag{B-17a}$$

(2) For $c \leqq T \leqq 2c$

$$\frac{\lambda(2c - T) + 1}{\lambda^2} + \frac{e^{-\lambda T}}{\lambda^2} \left(1 - 2e^{\lambda c}\right) . \tag{B-17b}$$

Combining Equations B-17, B-15, B-6, and B-3 for large values of t:

(1) For $0 \leqq T \leqq c$

$$s_1 = \frac{\lambda_5}{\lambda} + \frac{\lambda_6}{\lambda^2 c} \left\{ \left[1 + \tanh\left(\frac{\lambda c}{2}\right)\right] e^{-\lambda T} + \lambda T - 1 \right\} ; \tag{B-18a}$$

(2) For $c \leqq T \leqq 2c$

$$s_1 = \frac{\lambda_5}{\lambda} + \frac{\lambda_6}{\lambda^2 c} \left\{ \left[1 + \tanh\left(\frac{\lambda c}{2}\right)\right] e^{-\lambda T} - 2e^{-\lambda(T-c)} + (2c - T)\,\lambda + 1 \right\} . \tag{B-18b}$$

The differential equation describing the periodic behavior of the metabolic end products is

$$\frac{dp_1}{dt} + Dp_1 = c_1 \left[Dz_o - Ds_1 - \frac{ds_1}{dt} \right] . \qquad \text{(B-19)}$$

The last two terms on the right are small and are eliminated for mathematical convenience. Thus

$$\frac{dp_1}{dt} + Dp_1 \simeq c_1 Dz_o \ . \qquad \text{(B-20)}$$

Two experimental cases are important using the sawtooth forcing function since they permit ready solutions to Equation B-20. The first situation is for constant feed strength, z_o. For this case

$$p_1 = c_1 z_o \qquad \text{(B-21)}$$

within the limits of the validity of Equation B-20 and regardless of the functional nature of D.

For the case in which the forcing function is applied to the feed strength, z_o, at constant feed rate, the solution to Equation B-20 is very similar to that for Equation B-19. Letting

$$\lambda_7 = \left(c_1 Dz_o \right)_{min}$$

$$\lambda_7 + \lambda_8 = \left(c_1 Dz_o \right)_{max} , \qquad \text{(B-22)}$$

then the differential equation, B-20, becomes

$$\frac{dp_1}{dt} + Dp_1 = \lambda_7 + \frac{\lambda_8}{c} \left[H(c,t) \right] . \qquad \text{(B-23)}$$

Letting $t \rightarrow \infty$ the two solutions for Equation B-23 are:

(1) For $0 \leq T \leq c$

$$p_1 = \frac{\lambda_7}{D} + \frac{\lambda_8}{D^2 c} \left\{ \left[1 + \tanh \left(\frac{Dc}{2} \right) \right] e^{-DT} + DT - 1 \right\} ; \qquad \text{(B-24a)}$$

(2) For $c \leq T \leq 2c$

$$p_1 = \frac{\lambda_7}{D} + \frac{\lambda_8}{D^2 c} \left\{ \left[1 + \tanh \left(\frac{Dc}{2} \right) \right] e^{-DT} - 2e^{-D(T-c)} + (2c - T) D + 1 \right\} . \qquad \text{(B-24b)}$$

The response of total soluble COD, z_1, is given by the sum of Equations B-18 and B-24, since the relationship in Equation B-21 is incorporated in B-24, i.e., if D is a constant then $\lambda_8 = 0$.

APPENDIX C

STEADY STATE LABORATORY DATA[a]

Run No.	COD, mg/ℓ		Feed Rate, F ℓ/day	Cell Dilution Rate, D_c day^{-1}	MLSS mg/ℓ	MLVSS x_1 mg/ℓ	Substrate Removal Rate q_z lb COD/lb VSS-day	Sludge Volume Index mℓ/g
	Influent z_o	Soluble Effluent z_1						
65-21	357	31.5	2.4	0.030	3960	3540	0.27	160
	385	33.5	2.2	0.030	3965	3545	0.27	126
	370	29.0	2.2	0.031	4185	3750	0.25	
	364	26.5	2.2	0.031	4135	3710	0.26	107
		20.5	2.35	0.032	4380	3960	0.25	
	389		2.3	0.032	4560	4140		
	361	26.0	2.1	0.031	4720	4280	0.25	100
	357	23.0	2.5	0.032	4750	4250	0.25	120
		23.0	2.5	0.034	4960	4460	0.23	
	350	24.5	2.7	0.036	5160	4660	0.24	
	349	29.7	2.15	0.050	5060	4600	0.19	93
65-22	487	36.5	2.33	0.072	5560	4880	0.27	
	474	33.4	2.30	0.098	5160	4550	0.28	128
		34.1	2.22	0.053	5260	4610	0.28	125
	514	33.7	2.16	0.031	5320	4680	0.28	133
	502	32.4	2.15	0.028	5440	4820	0.26	131
	472	30.8	2.20	0.032	5500	4900	0.25	118
	468	27.6	2.15	0.035	5890	5260	0.23	122
	451	35.6	2.10	0.035	5610	5030	0.22	
	431	28.1	(2.10)	0.032	5550	5040	0.21	126
	454	40.0	2.05	0.040	5550	4990	0.21	
		42.4	2.13	0.043	5880	5290	0.21	138
	506	48.0	2.34	0.036	5620	5040	0.27	146
	478	45.0	2.20	0.032	5460	4900	0.24	
65-23	387	51.0	2.20	0.045	3320	3060	0.30	96
	382	60.0	2.20	0.065	3330	3020	0.29	87
	378	56.0	2.40	0.069	3010	2750	0.35	90
	387	50.5	(2.25)	0.053	3320	3000	0.32	
		57.0	2.20	0.055	3330	3010	0.30	
	387	52.4	2.24	0.045	3320	3020	0.31	87
	373	50.7	2.30	0.045	3510	3150	0.29	85
	388	49.5	2.30	0.050	3420	3120	0.31	91
65-24	(417)	44.3	2.3	0.090	2700	2480	0.43	104
		37.3	2.6	0.096	2720	2680	0.46	104
	396	34.6	2.6	0.102	2760	2520	0.47	112
	396	34.6	2.4	0.105	2740	2520	0.43	124
		33.2	2.6	0.100	2840	2630	0.45	130
65-25		20.0	2.8	0.055	4260	3760	0.19	157
	220	16.2	2.3	0.043	4360	3660	0.16	
		15.2	2.6	0.038	4220	3760	0.18	
65-26	605	68.0	2.50	0.113	4220	3640	0.46	76
			2.60	0.116	3860	3520	0.46	124
		62.0	2.50	0.113	3820	3400	0.43	89
	594	65.5	2.50	0.115	3760	3380	0.49	80
		75.5	2.40	0.100	3780	3440	0.39	
	580	71.0	2.40	0.112	3780	3380	0.45	90
		76.0	2.50	0.106			0.46	
		72.5	2.74	0.105	3940	3520	0.51	89
	590	74.0	2.25	0.116	3900	3520	0.41	
	586	78.0	2.70	0.124	4120	3820	0.43	78
65-27		22.8	2.40	0.025	4120	3480	0.16	90
	200	24.6	2.30	0.032	4100	3480	0.15	95
	220	21.8	2.40	0.034	4000	3440	0.17	102
	216	23.2	2.50	0.034	3960	3460	0.17	106
			2.40	0.022	3780	3320	0.17	
65-27	253	34.7	2.6	0.157	1820	1480	0.48	
			2.4	0.144	1760	1440	0.41	
	255		(2.5)	0.139	1600	1360	0.44	
			2.5	0.138	1340	1280	0.47	
			2.45	0.118	1380	1200	0.49	515
		43.5	2.65	0.102	1520	1460	0.48	
			2.6	0.115	1560	1480	0.47	512
		35.7	2.4	0.118	1500	1460	0.36	
65-28	424	33.7	2.45	0.038	6440	5760	0.21	109
		35.4	2.7	0.053	6820	6120	0.21	
		36.0	2.94	0.045	6560	5880	0.24	107
	409	36.0	2.35	0.020	6700	6020	0.18	
	414	35.6	2.4	0.044	6780	6180	0.18	110
	423	35.8	2.2	0.041	6440	5820	0.18	

[a] Reactor volume = 0.8 liter.

STEADY STATE LABORATORY DATA (Continued)

Run No.	COD, mg/ℓ		Feed Rate, F ℓ/day	Cell Dilution Rate, D_c day^{-1}	MLSS mg/ℓ	MLVSS x_1 mg/ℓ	Substrate Removal Rate q_z lb COD/lb VSS-day	Sludge Volume Index mℓ/g
	Influent z_0	Soluble Effluent z_1						
65-29	238	31.8	2.45	0.120		1260	0.53	
	239	31.2	2.4	0.117	1500	1400	0.48	
	239	28.9	2.5	0.092	1400	1280	0.51	550
			2.3	0.082	1520	1400		
65-29		44.0	2.6	0.084	2680	2560	0.47	
			2.5	0.111	2480	2440	0.47	315
	412	42.9	2.3	0.110			0.44	
			2.3		2440	2270	0.47	
65-30	547	38.6	2.5	0.190	2820	2570	0.62	92
		42.0	2.47	0.183	2850	2610	0.57	98
		41.4	2.43	0.170	2710	2500	0.59	
	569	40.0	2.5	0.189	2690	2450	0.65	119
		39.5	2.5	0.166	2850	2610	0.61	
	605	42.5	2.5	0.161	2970	2720	0.64	111
		45.6	2.5	0.146	2910	2670	0.65	
		46.0	2.4	0.150	3010	2790	0.60	
65-31	400	30.5	2.6	0.203	2000	1840	0.66	280
		35.3	2.66	0.229	1760	1620	0.76	330
		35.2	2.54	0.190	1960	1800	0.65	
	412	33.3	2.5	0.176	2240	2030	0.57	308
		32.2	2.4	0.166	2290	2050	0.55	
	420	30.0	2.4	0.182	2350	2110	0.53	187
		34.1	2.3	0.177	2370	2120	0.52	
65-32		26.2	2.5	0.258	1490	1360	0.52	
	252	27.5	2.5	0.287	1430	1290	0.59	364
		25.7	2.3	0.220	1490	1350	0.50	
	255	23.2	2.3	0.239	1610	1450	0.47	335
		24.7	2.55	0.212	1670	1500	0.48	
		23.4	2.40	0.186	1440	1240	0.55	
65-33		33.5	2.45	0.191	910	880	0.77	
	252	30.8	2.4	0.206	1010	1010	0.67	753
		28.3	2.46	0.203			0.73	
	255	27.7	2.4	0.198	1030	940	0.72	
		26.5	2.4	0.234	945	845	0.81	666
	259	23.5	2.3	0.117	915	815	0.82	
65-34	798	163	2.4	0.790	1380	1230	1.55	587
	830	226	2.4	0.671	1240	1140	1.52	
			(2.4)	0.476			1.79	
	827	243	2.5	0.424	1060	960	1.91	660
	827	216	2.65	0.405	1430	1270	1.60	
		264	2.4	0.506	1430	1250	1.36	630
	812	239	2.5	0.643	1130	980	1.89	
	828	298	2.44	0.695	1060	935	1.74	688
			2.34	0.385			1.79	
65-35	618	129	2.4	0.374	2150	1690	0.86	88
		132	2.4	0.393	2040	1650	0.89	
	609	137	2.4	0.511	1290	1090	1.30	101
	603	126	2.5	0.271	1690	1470	1.03	
	600	132	2.4	0.320	1710	1520	0.95	100
	618	131	2.45	0.281	2090	1870	0.79	
			(2.4)	0.297			0.83	
	633	113	(2.4)	0.348	1940	1750	0.90	268
65-35	624	201	2.4	0.864	990	950	1.56	810
		227	2.4	0.814	850	810	1.57	
	643	218	2.4	0.881	690	640	2.03	796
		264	2.4	0.801	705	645	1.80	
65-36	429	139	2.55	0.633	665	605	1.50	255
		135	2.5	0.637	725	665	1.36	
	430	143	2.3	0.809	615	565	1.44	650
		139	2.4	0.688	575	535	1.60	
	418	116	2.45	0.554	675	610	1.55	518
		116	2.45	0.451	615	520	1.69	
65-37		66	2.4	0.381	610	555	0.76	
	229	71	2.1	0.351	725	645	0.60	276
		67	2.4	0.290	680	610	0.68	
	218	70	2.35	0.256	680	620	0.64	397
		72	2.4	0.408	640	560	0.72	
65-38		64.3	2.35	0.239	2880	2640	0.84	
	867	78.3	2.34	0.264	2720	2440	0.91	269
		137	2.66	0.248	2640	2380	0.98	
	812	123	2.4	0.228	2640	2560	0.84	
		98	2.5	0.249	2640	2520	0.92	348
		83	2.3	0.268	2460	2320	0.94	

STEADY STATE LABORATORY DATA (Continued)

Run No.	COD, mg/ℓ		Feed Rate, F ℓ/day	Cell Dilution Rate, D_c day^{-1}	MLSS mg/ℓ	MLVSS x_1 mg/ℓ	Substrate Removal Rate q_z lb COD/lb VSS-day	Sludge Volume Index mℓ/g
	Influent z_o	Soluble Effluent z_1						
65-39	606	63.5	2.4	0.181	2400	2200	0.75	183
		60.6	2.35	0.169	2530	2310	0.70	
	599	69.9	2.8	0.188	2440	2300	0.82	246
		67.9	2.5	0.162	2600	2380	0.71	
		67.6	2.3	0.166	2660	2380	0.66	
	627	58.9	2.4	0.198	2520	2280	0.73	325
		59.0	2.4	0.202	2520	2340	0.71	
65-39	204	27.5	3.0		1060	1000	0.66	
	207	27.8	2.4	0.166	990	950	0.56	445
65-40	413	44.7	2.5	0.202	1690	1580	0.73	148
		44.7	2.5	0.175	1690	1560	0.74	
	417	44.9	2.5	0.181	1770	1660	0.70	226
		45.8	2.4	0.163	1820	1670	0.66	
		45.3	2.5	0.153	1870	1620	0.71	
	412	48.4	2.35	0.156	1940	1780	0.61	392
		41.8	2.4	0.147	2000	1830	0.62	
	420	41.4	2.4	0.144	1930	1710	0.66	352
		39.3	2.7	0.118	1970	1790	0.72	
		37.1	2.7	0.145	1990	1850	0.70	
65-41	210	34.3	2.45	0.190	740	680	0.82	243
		37.2	2.45	0.129	870	800	0.68	
	208	35.0	2.45	0.114	950	880	0.63	210
		35.1	2.45	0.162	840	765	0.72	
		34.9	2.35	0.157	950	855	0.62	
65-41	418	53.2	2.4	0.185	1535	1410	0.79	
		54.2	2.4	0.210	1395	1285	0.87	244
		55.0	2.4	0.147	1450	1390	0.80	
66-S-1	213	14.3	2.44	0.040	3850	2990	0.20	
		13.8	2.40	0.041	3820	3000	0.19	
		14.6	2.20	0.043	3930	2900	0.19	
		13.3	2.58	0.042	4150	3260	0.19	
66-S-2	217	17.6	2.44	0.211	1340	1110	0.53	
		16.5	2.71	0.186	1200	1000	0.68	
		14.6	2.20	0.170	1338	1118	0.50	
		17.0	2.68	0.160	1340	1140	0.60	
66-S-3	418		2.26					
		30.2	2.4	0.400	1270	1120	1.04	
		27.1	2.35	0.512	1000	884	1.30	
		27.2	2.37	0.580	1040	920	1.26	
66-S-4	622	58.3	2.45					
		53.0	2.70	0.97	815	755	2.58	
		53.0	2.57	0.76	940	905	2.02	
		48.5	2.55	1.07	815	755	2.42	
66-S-5	184	12.8	2.09	0.039	2370	1810	0.25	101
		11.4	2.23	0.041	2380	1840	0.26	103
		11.5	2.23	0.041	2330	1810	0.27	99
66-S-6	406	24.2	2.51	0.184	2310	2060	0.58	130
		22.9	2.59	0.190	2320	2040	0.62	142
	422	26.4	2.24	0.171	2370	2090	0.54	118
		21.7	2.19	0.187	2210	1960	0.56	150
66-S-8	412	40.8	3.85	0.94	1020	920	1.95	147
		39.2	3.85	0.82	925	845	2.12	65
		39.6	4.12	1.37	725	645	2.78	940
		41.8	4.12	1.26	785	570	3.14	1100
66-01	236		2.48		990	920	0.73	580
		19.3	2.3		1050	970	0.66	
			2.3		1020	940	0.66	
		17.5	2.4					
		17.4	2.4		1070	990	0.66	
					1040	940	0.70	
66-03	208		2.6		1510	1390	0.44	
			2.4				0.42	
		15.8	2.4		1410	1310	0.45	
		15.3	2.35		1420	1300	0.44	
	214	14.4	2.35		1490	1350	0.43	463

[a] Reactor volume = 0.8 liter.

STEADY STATE LABORATORY DATA (Continued)

Run No.	COD, mg/ℓ		Feed Rate, F ℓ/day	Cell Dilution Rate, D_c day^{-1}	MLSS mg/ℓ	MLVSS x_1 mg/ℓ	Substrate Removal Rate q_s lb COD/lb VSS-day	Sludge Volume Index mℓ/g
	Influent z_o	Soluble Effluent z_1						
66-04		16.9	2.5		1110	1040	0.48	
		21.6	2.5		1400	1290	0.47	
		16.4	2.5		1410	1330	0.47	631
	213		2.45		1430	1340	0.46	
	220	16.0	2.45		1510	1370	0.46	417
		12.6	2.6		1570	1450	0.46	
66-05	214	14.2	2.4		2290	1900	0.31	144
		14.2	2.4		2200	1830	0.33	
		14.6	2.43		2200	1870	0.32	
		12.9	2.43		2320	1970	0.31	
	212	14.3	2.43		2470	2060	0.29	239
66-06	419	21.6	2.18		1860	1710	0.63	
		29.5	2.4				0.68	
		25.5	2.4		1900	1730	0.68	
		20.3	2.4		2010	1820	0.66	
66-07		23.3	2.4		2890	2510	0.47	
		27.4	2.53		3340	2860	0.43	
		25.9	2.53		2940	2570	0.48	
	420	23.6	2.53		2960	2600	0.48	247
66-25	222	12.8	2.52		960	820	0.80	210
		13.3	2.52		850	750	0.88	235
		18.0	2.53		690	620	1.04	160
	215	16.3	2.6		640	580	1.12	300
66-25		17.0	2.5		480	430	1.44	165
		17.9	2.3		420	360	1.60	
		17.9	2.35		425	395	1.53	235
	218		2.4		465	370		345
		20.4	2.5		445	350	1.77	
		18.3	2.5		370	340	1.84	
66-32	190	21.5	4.92		350	320	3.46	315
		26.4	5.00		350	325	3.18	370

APPENDIX D

PILOT PLANT DAILY OPERATIONAL DATA OBTAINED IN THIS STUDY[a]

Date	COD, mg/ℓ		BOD, mg/ℓ		Feed Rate, F gpd	Waste Rate gpd	$\frac{MLVSS}{MLSS}$ %	MLVSS x_1 mg/ℓ	Effluent MLVSS mg/ℓ	Cell Dilution Rate, D_c day^{-1}	Substrate Removal Rate q_z lb COD/lb VSS-day	Sludge Volume Index mg/ℓ
	Influent z_0	Soluble Effluent z_1	Influent	Soluble Effluent								
Pilot Plant Unit No. 2												
5-26-66	221	52	118	13	4754	462	82	620	9	0.93	2.29	132
5-27-66	210	47	115	11	4754	483	84	580	12	1.00	2.38	101
5-28-66	240	43			4150	470	83	628	14	0.98	2.30	
5-29-66	243	41			4150	470	87	656	15	0.98	2.26	
5-30-66	247	46			4150	470	88	464	30	1.25	3.19	
5-31-66	278	59			4350	470	86	432	44	1.52	3.88	
6- 1-66	285	62			4750	470	86	372	36	1.58	5.02	110
6- 2-66	247	42			4750	470	86	287	40	1.90	5.92	
6- 3-66	257	58			4750	360	82	432	16	0.93	3.90	
6- 6-66	299	58			4750	360	83	516	13	0.84	3.93	258
6- 7-66	266	45			4750	430	85	612	17	0.97	3.04	291
6- 8-66	263	45	147	8	4750	430	87	520	23	0.75	3.52	402
6- 9-66	245	45			4750	430	89	380	43	1.63	4.46	911
6-10-66	243	45			4750	430	91	368	35	1.50	4.55	718
6-11-66	256	40			4750	430	87	440	52	1.18	4.13	
6-12-66	269	41			4750	430	87	444	60	1.79	4.32	
6-13-66	263	44			4750	430	86	360	63	2.11	5.13	
6-14-66	266	44			4750	430	87	372	37	1.55	5.03	
6-15-66	255	44			4750	430	85	452	30	1.28	3.96	
6-16-66	248	36			4750	430	89	516	23	1.11	3.47	190
6-17-66	255	43			4750	430	82	636	28	1.10	2.81	65
6-18-66	249	40			4750	430	85	665	24	1.04	2.64	
6-19-66	246	39			4750	430	86	600	30	1.15	2.91	
6-20-66	264	42			4750	430	85	600	44	1.18	3.13	
6-21-66	235	39			4750	430		704		1.16	2.69	
6-22-66	211	57	120	>11	4750	430		666		0.97	2.08	
Pilot Plant Unit No. 3												
5-26-66	221	32	118	4	1440	60	82	1520	8	0.119	0.32	395
5-27-66	210	30	115	3	1440	60	82	1492	10	0.124	0.31	
5-28-66	240	36			1260	60	82	1584	13	0.124	0.29	
5-29-66	243	34			480	60	83	1496	11	0.113	0.13	
5-30-66	247	33			480	60	84	1352	13	0.114	0.14	
5-31-66	278	36			720	60	82	1180	10	0.116	0.26	
6- 1-66	285	27			1440	60	82	1148	13	0.134	0.69	
6- 2-66	247	32			1440	60	82	1080	13	0.136	0.51	
6- 3-66	257	46			1440	60	84	1388	13	0.129	0.39	
6- 6-66	299	42			1440	60	83	1264	20	0.145	0.52	92
6- 7-66	266	42			1440	60	84	1180	27	0.164	0.48	85
6- 8-66	263	39	147	5.5	1440	60	84	1024	23	0.159	0.56	90
6- 9-66	245	39			1440	60	83	1069	32	0.182	0.49	94
6-10-66	243	45			3600	144	87	1088	31	0.235	1.16	200
6-11-66	256	38			3600	144	87	1004	89	0.806	1.46	
6-12-66	269	32			2160	144	86	1012	96	0.593	1.08	
6-13-66	263	33	120	3	2160	144	86	860	19	0.391	1.03	270
6-15-66												
6-16-66	248	44	110	5	2160	144	85	848	22	0.350	0.93	380
6-17-66	255	50	115	7	2160	144	87	912	23	0.346	0.86	200
6-18-66	249	46			2160	144	85	940	21	0.344	0.91	
6-19-66	246	51			2160	144	83	850	47	0.473	0.97	
6-20-66	264	56			2160	144	85	775	44	0.478	1.13	
6-21-66	235	46			2160	144	84	732	49	0.495	0.99	
6-22-66	211	42	120	11	2160	144	84	704	28	0.400	0.92	
6-24-66	246	46			2160	144	83	660	23	0.380	1.19	

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[a] V = 552 gallons.

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